Super-Efficiency DEA Approach for Optimizing Multiple Quality Characteristics in Parameter Design

Abbas Al-Refaie, University of Jordan, Jordan

ABSTRACT

This paper proposes an efficient approach for optimizing the multiple quality characteristics (QCHs) in manufacturing applications on the Taguchi method using the super efficiency technique in data envelopment analysis (DEA). Each experiment in Taguchi’s orthogonal array (OA) is treated as a decision making unit (DMU) with multiple QCHs set as inputs or outputs. DMU’s efficiency is evaluated then adopted as a performance measure to identify the combination of optimal factor levels. Three real case studies were employed for illustration in which the proposed approach provided the largest total anticipated improvements in multiple QCHs among other techniques such as principal component analysis (PCA) and DEA based ranking (DEAR) approach. Analysis of variance is finally employed to decide significant factor effects and to predict performance.

Keywords: DEA, Decision Making Unit, Optimization, Super Efficiency, Taguchi Method

INTRODUCTION

The overall goal of robust design is to find settings of the controllable factors so that the QCH is least sensitive to variations in the noise variables, while still yielding an acceptable mean level of the QCH. A particularly cost-effective approach is the robust design introduced by Taguchi (1991), which aims for finding the optimal settings of control factors to make the product or process insensitive to noise factors. Typically, the QCH is divided into three main types; the smaller-the-better (STB), the nominal-the-best (NTB), and the larger-the-better (LTB) type QCHs. In the Taguchi method, an orthogonal array (OA) is utilized to reduce the number of experiments under permissive reliability. Signal-to-noise (S/N) ratio is then adopted to optimize performance. Although this method has been widely applied, it is usually efficient for only optimizing a single QCH. Recently, optimization of multiple QCHs in the Taguchi method was faced by several studies (Tong et al., 1997; Lin & Lin, 2002; Jean & Wang, 2006; Al-Refaie et al., 2008; Al-Refaie et al., 2009).

Data envelopment analysis (DEA) developed by Charnes et al. (1978) is a fractional mathematical programming technique widely

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used for measuring the performance of homogeneous decision making units (DMUs) with multiple inputs and multiple outputs at organizational level; such as cities, hospitals, and universities. The most popular DEA technique is the CCR model developed by Charnes, Cooper, and Rhodes (1978). Assuming there are \( n \) DMUs each with \( m \) inputs and \( s \) outputs to be evaluated, then the CCR model measures the relative efficiency of each DMU once by comparing it to a group of the other DMUs that have the same set of inputs and outputs. Hence, there are \( n \) optimizations needed. The relative efficiency, \( E_o \), of DMU \( o \) with inputs of \( X_{io} (i = 1, \ldots, m) \) and outputs of \( Y_{ro} (r = 1, \ldots, s) \) is evaluated by CCR model as (Charnes et al., 1994)

\[
E_o = \text{Max } \theta = \frac{\sum_{r=1}^{s} U_r Y_{ro}}{\sum_{i=1}^{m} V_i X_{io}}
\]

subject to

\[
E_j = \frac{\sum_{r=1}^{s} U_r Y_{rj}}{\sum_{i=1}^{m} V_i X_{ij}} \leq 1, \quad j = 1, \ldots, o, \ldots, n
\]

\[
U_r \geq 0, \quad r = 1, 2, \ldots, s
\]

\[
V_i \geq 0, \quad i = 1, 2, \ldots, m
\]

(1)

where \( Y_{rj} \) is the \( r \)th output for DMU \( j \), \( X_{ij} \) is the \( i \)th input for DMU \( j \), \( U_r \) is the weight given to the \( r \)th output, \( V_i \) is the weight assigned to the \( i \)th input, and \( \theta \) is a scalar. The objective function is to maximize the efficiency of DMU being evaluated, DMU \( o \), subject to the relative efficiency, DMU \( j \) (\( j = 1, \ldots, n \)) is less or equal to one with positive virtual weights. The CCR model identifies DMU \( o \) efficient as if \( E_o \) equals one. Otherwise, DMU \( o \) is identified as CCR-inefficient. Obviously, the CCR model is nonlinear, which can be converted into a linear programming problem by considering the “input-oriented” CCR model expressed as

\[
E_o = \text{Max } \theta = \sum_{r=1}^{s} U_r Y_{ro}
\]

subject to

\[
\sum_{i=1}^{m} V_i X_{io} = 1,
\]

\[
\sum_{i=1}^{m} U_i Y_{io} - \sum_{r=1}^{s} V_r Y_{ro} \leq 0, \quad j = 1, 2, \ldots, o, \ldots, n
\]

\[
U_r \geq 0, \quad r = 1, 2, \ldots, s
\]

\[
V_i \geq 0, \quad i = 1, 2, \ldots, m
\]

(2)

Khouja (1995) adopted the CCR model in the comparison between efficient robot technologies. However, Baker, and Talluri (1997) investigated the robot selection problem in Khouja’s study and concluded that the CCR model provides misleading efficiency scores by allowing for complete weight flexibility and thus results in identifying a DMU with an unrealistic weighing scheme to be efficient. In addition, when the efficiency scores for some DMUs are equal to one, the CCR model fails to discriminate among efficient DMUs. Hence, expert judgment is finally needed to decide optimal factor levels, which increase the uncertainty in the decision making process.

Contrary to the CCR model, the super efficiency technique (Banker et al., 1989; Andersen & Petersen, 1993) in DEA increases discrimination among efficient DMUs by allowing efficiency scores taking values greater than one. This technique compares DMU \( o \) with a linear combination of other DMUs of the sample while excluding DMU \( o \). This only affects the efficiency scores of the extreme efficient DMUs, which consequently can obtain an efficiency score greater than one. Let \( E'_o \) denotes the obtained efficiency score of DMU \( o \) obtained using super efficiency technique evaluated by solving the following dual model:

\[
E'_o = \text{Min } \theta
\]

subject to

\[
\sum_{j=1, j \neq o}^{n} \lambda_j X_j \leq \theta X_o
\]

\[
\sum_{j=1, j \neq o}^{n} \lambda_j Y_j \geq Y_o
\]

\[
\lambda_j \geq 0
\]

(3)

where \( X_j \) is an \( m \)-dimensional input vector and \( Y_j \) is an \( s \)-dimensional output vector, while \( \theta \) is a scalar. The \( E'_o \) values are then used for comparing and analyzing the performance of the \( n \) DMUs. This technique is utilized in optimizing
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