Chapter 16
Granular Computing in Formal Concept

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ABSTRACT

Granular computing has permeated through the field of formal concept; it is another new and rapid developmental aspect of formal concept. In this chapter, we’ll regard supremum semisublattice, infimum semisublattice and sublattice as “granule”. When a set of granules covers the lattice, “granular space” is called on the concept lattice. We study mainly granular spaces generated by ideal-filter congruence relations and tolerance relations. We emphasize properties of these granular spaces and generating methods of these granular spaces. By our viewpoint to study granular computing in formal concept, we find out that it shows profound relation and essence of various sublattices.

BACKGROUND

Granular computing and formal concept as new ideas of the field of knowledge discovery have been received more attention about their relation. Granular computing was first proposed by L.A. Zadeh in 1998. Much affection had been generated in various scientific fields in recent decade. Formal concept analysis was firstly proposed by Rudolf Wille, a German professor, in 1982. The most basic elements of human thinking—concept and their hierarchy was explored, analyzed and researched with mathematical method by this theory. It is a branch of lattice theory. In recent years, granular computing has permeated through the field of formal concept as well. The granules and their hierarchy which were generated by equivalence relations in formal concept were researched by Y.Y. Yao & J.T. Yao in 2002 by Du W.L & Miao D.Q in 2005 and by Z. Zheng, H. Hu, Z. Shi. The properties of granules generated by equivalence...
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relations in concept lattice were researched profoundly. Some further works were done by us on granular computing in formal concept, too. The speech was given which is titled by “Granular Spaces Generated by Congruence Relations in Concept Lattice” in the Assembly Discussions on “granular computing” of the 5th Chinese Conference on Rough Set and Software Computation in 2005. It aroused many echoes. Under such background, we more expand the research about granular spaces generated by congruence relations, and promote the research about granular spaces generated by ideal-filter and also advance the research about granular spaces generated by tolerance relation.

1. BASIC DEFINITION

Definition 1.1

Let \( U \) be a set of objects, \( M \) is a set of attributes, and \( I \subseteq U \times M \) is a relation between \( U \) and \( M \). \( \theta \) is called a formal context (context for short). Let \( A \) is a subset of \( U \) and \( B \) is a subset of \( M \), we define two functions \( f(A) \) and \( g(B) \) as:

\[
f(A) = \{ m \in M \mid \forall u \in A, (u, m) \in I \}
\]

\[
g(B) = \{ u \in U \mid \forall m \in B, (u, m) \in I \}
\]

then, \((A, B)\) is called a formal concept (concept for short) on context \((U, M, I)\) with \( f(A) = B \) and \( g(B) = A \), where \([a]^q = [(a^\prime)_q, a^r] , B \subseteq M \). \( A \) is called extent of the concept, \( B \) is called intent of the concept. The set of all concepts on \((U, M, I)\) is denoted by \( \mathfrak{B}(U, M, I) \)

If \( A, A_1, A_2 \subseteq U \), \( B, B_1, B_2 \subseteq M \), there are some properties, which will be used in this chapter, as follows:

1) \( A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2) \) 1') \( B_1 \subseteq B_2 \Rightarrow g(B_2) \subseteq g(B_1) \)

2) \( f(A) \subseteq f(B) \) (2') \( B \subseteq f(g(B)) \)

3) \( f(A) = f(g(f(A))) \) (3') \( g(B) = g(f(g(B))) \)

4) \( f(A) \cup f(A) = f(A) \cap f(A) \) (4') \( g(B) \cup g(B) = g(B) \cap g(B) \)

Note that 4) and 4') can be extended as follows: given by an index set \( T \), if for each \( t \in T \), \( A_t \subseteq U \) and \( B_t \subseteq M \), then

\[
f(\bigcup_{t \in T} A_t) = \bigcap_{t \in T} f(A_t)\quad \text{and} \quad g(\bigcup_{t \in T} B_t) = \bigcap_{t \in T} g(B_t).
\]

On the other hand, by 2) and 2'), for any subset \( A \) of \( U \), \( \big( g \left( f(A) \right), f(A) \big) \) must be a concept and for any subset \( B \) of \( M \), \( \big( g(B), f \left( g(B) \right) \big) \) must be a concept as well. Especially, for an object \( u \), \( \big( g \left( f(u) \right), f(u) \big) \)