A New Hybrid Inexact Logarithmic-Quadratic Proximal Method for Nonlinear Complementarity Problems

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ABSTRACT

In this paper, the authors present and analyze a new hybrid inexact Logarithmic-Quadratic Proximal method for solving nonlinear complementarity problems. Each iteration of the new method consists of a prediction and a correction step. The predictor is produced using an inexact Logarithmic-Quadratic Proximal method, which is then corrected by the Proximal Point Algorithm. The new iterate is obtained by combining predictor and correction point at each iteration. In this paper, the authors prove the convergence of the new method under the mild assumptions that the function involved is continuous and monotone. Comparison to another existing method with numerical experiments on classical NCP instances demonstrates its superiority.

Keywords: Equilibrium, Nonlinear Complementarity Problem, Nonlinear Programming, Proximal Point, Step Size

INTRODUCTION

The nonlinear complementarity problem (NCP) is to determine a vector \( x \in \mathbb{R}^n \) such that

\[
x \geq 0, \quad F(x) \geq 0 \quad \text{and} \quad x^T F(x) = 0, \quad (1)
\]

where \( F : \mathbb{R}^n \to \mathbb{R}^n \) is a nonlinear mapping. Throughout this paper we assume that \( F \) is continuous and monotone with respect to \( \mathbb{R}_+^n \), and the solution set of (1) is nonempty. NCP has many important applications in engineering, economics, military operations planning, finance, medical treatment, supply chain management etc. (Auslender & Haddou, 1995; Das, 2009; Castagnoli & Favero, 2008; Ferris & Pang, 1997; Harker & Pang, 1990; Yan & Wang, 1997). Many numerical methods for solving NCP have been developed (Auslender & Haddou, 1995; Auslender et al., 1999; Bnouhachem & Noor, 2001; Burachik & Svaiter, 2001; Censor et al., 1994; Eckstein, 1998; Fischer, 1997; Guler, 1991; Iusem, 1998; Pang, 1995; Qi & Yang, 2002; Sun & Qi, 1999; Zhou, 2009).

DOI: 10.4018/joris.2010070101
NCP can be alternatively formulated as finding the zero point of an appropriate maximal monotone operator

\[ T(x) := F(x) + N_{R^n_+}(x), \tag{2} \]
i.e., finding \( x^* \in R^n_+ \) such that \( 0 \in T(x^*) \), where \( N_{R^n_+}(\cdot) \) is the normal cone operator to \( R^n_+ \) defined by

\[ N_{R^n_+}(x) := \begin{cases} \{z \in R^n : (x^* - x, z) \leq 0, \forall x^* \in R^n_+\} & \text{if } x \in R^n_+, \\ \emptyset & \text{otherwise.} \end{cases} \tag{3} \]

A well known method to find the zero point of a maximal monotone operator \( T \) is the proximal point algorithm (PPA), which starts with any vector \( x_0 \in R^n_+ \) and \( c \geq c > 0 \) and iteratively updates \( x^{k+1} \) conforming the following problem:

\[ \text{(PPA)} \quad 0 \in c_k T(x) + (x - x^k). \tag{4} \]

In order to obtain the new point \( x^{k+1} \), the subproblem (1.4) of PPA is equivalent to the following variational inequality problem:

Find \( x \in R^n_+ \) such that

\[ (x^* - x, x - x^*) \geq 0, \forall x^* \in R^n_+. \tag{5} \]

Let \( x^{k+1} \) be the solution of (5), the new iterate \( x^{k+1} \) of the PPA can be expressed as the following equivalent recursion form:

\[ x^{k+1} = P_{R^n_+}[x^k - c_k F(x^{k+1})], \tag{6} \]

where \( P_{R^n_+} \) denotes the projection on \( R^n_+ \).

The ideal form (6) of PPA is often impractical since in many cases solving problem (5) often computationally expensive or even impossible to solve exactly (Auslender et al., 1999).

Rockafellar (1976) established the fundamental convergence analysis for the Approximate Proximal Point Algorithm (APPA) to a general maximal monotone operator. The new iterate \( x^{k+1} \) of Rockafellar’s APPA is set to satisfy

\[ \|x^{k+1} - x^{k+1}\| \leq \nu_k, \quad \sum_{k=0}^{\infty} \nu_k < +\infty. \tag{7} \]

Since \( x^{k+1} \) is unknown, some upper bounds of \( \|x^{k+1} - x^k\| \) need to be estimated in order to implement the APPA.

A number of papers have focused on generalization of PPA by replacing the linear term \( x - x^k \) with some nonlinear functions \( r(x, x^k) \) (Auslender & Haddou, 1995; Solodov & Svaiter, 1999; Teboulle, 1997). Auslender et al. (1999) improved PPA (4) by replacing the linear term \( x - x^k \) with

\[ x - (1 - \mu)x^k - \mu X_k x^{-1}, \tag{8} \]

where \( \mu \in (0, 1) \) is a given constant, \( X_k = \text{diag}(x_{j1}, x_{j2}, \ldots, x_{jn}) \), and \( x^{-1} \) is an \( n \)-vector whose \( j \)th elements is \( 1 / x_j \). In particular, at the \( k \)th iteration, solving (1) by the LQP method is equivalent to finding the positive solution to the following system of nonlinear equations (LQP system for convenience), which needs to be solved exactly:

\[ c_k F(x) + x - (1 - \mu)x^k - \mu X_k x^{-1} = 0. \tag{9} \]

Xu et al. (2006) introduced LQP based prediction-correction methods to avoid having
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