Stochastic Learning for SAT-Encoded Graph Coloring Problems

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ABSTRACT

The graph coloring problem (GCP) is a widely studied combinatorial optimization problem due to its numerous applications in many areas, including time tabling, frequency assignment, and register allocation. The need for more efficient algorithms has led to the development of several GC solvers. In this paper, the authors introduce a team of Finite Learning Automata, combined with the random walk algorithm, using Boolean satisfiability encoding for the GCP. The authors present an experimental analysis of the new algorithm's performance compared to the random walk technique, using a benchmark set containing SAT-encoding graph coloring test sets.

Keywords: Combinatorial Optimization, Graph Coloring Problems, Learning Automata, Random Walk Algorithm, SAT

1. INTRODUCTION

In the graph coloring problem (GCP) an undirected graph $G(V, E)$ is given, where $V$ is a set of vertices, and $E$ is a set of pairs of vertices called edges. We call a $k$-coloring of $G$, a mapping $C : V \rightarrow \{1 \ldots k\}$ such that if $C(p) = C(q)$ then $(p, q) \notin E$. The set $\{1 \ldots k\}$ is the set of colors. There exist two variants of this problem. In the optimization variant, the goal is to find the chromatic number $X(G)$, which is the minimal $k$ for which there exists a $k$-coloring of $G$. In the decision variant, the question is to decide whether for a particular number of colors, a coloring of $G$ exists. All these problems are known to be NP-complete, so it is unlikely that a polynomial-time algorithm exists that solves any of these problems.

In this paper the focus is on the decision variant of the GCP. Encoding the GCP as a Boolean satisfiability problem (SAT) and solving it using efficient SAT algorithms has caused considerable interest. The SAT problem, which is known to be NP-complete (Cook, 1971), can be defined as follows. A propositional formula $F = \bigwedge_{j=1}^{m} C_j$ with $m$ clauses and $n$ Boolean variables is given. A Boolean variable is a variable that can take one of the two values, True or False. Each clause $C_j$, in turn, has the form:

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\[ C_j = \left( \bigvee_{k \in I_j} \overline{x}_k \right) \lor \left( \bigvee_{l \in J_j} \overline{x}_l \right), \]

where \( I_j, \overline{I}_j \subseteq \{1, \ldots, n\} \), \( I_j \cap \overline{I}_j = \emptyset \), and \( \overline{x}_j \) denotes the negation of \( x_j \). The task is to determine whether the propositional formula \( \Phi \) evaluates to True. Such an assignment, if it exists, is called a satisfying assignment for \( \Phi \), and \( \Phi \) is called satisfiable. Otherwise, \( \Phi \) is said to be unsatisfiable. Most SAT solvers use a Conjunctive Normal Form (CNF) representation of the propositional formula. In CNF, the formula is represented as a conjunction of clauses, where each clause is a disjunction of literals, and a literal is a Boolean variable or its negation. For example, \( P \lor Q \) is a clause containing the two literals \( P \) and \( Q \). This clause is satisfied if either \( P \) is True or \( Q \) is True. When each clause in \( \Phi \) contains exactly \( k \) literals, the resulting SAT problem is called \( k \)-SAT.

The paper is organized as follows. In Section 2, we review various algorithms for solving GCP, as well as satisfiability algorithms for solving SAT-encoded GCP. Section 3 explains the basic concepts of Learning Automata (LA) and introduces our new LA based approach to graph coloring. In Section 4, we look at the results from testing the new approach and draw some conclusions. Finally, in Section 5 we present a summary of the work.

### 2. RELATED WORK

#### 2.1 Graph Coloring Algorithms

The GCP is a well-known problem in combinatorial optimization. It is among the earliest problems proved to be NP-Complete (Karp, 1991). A number of exact solution approaches based on integer programming models have been proposed (Brown, 1972; Brelaz, 1979; Swell, 1996; Hansen, Labbe, & Schindl, 2005; Mendez-Diaz & Zabala, 2008). Still, compared to the variety of graph coloring methods proposed in the field as a whole, the number of such algorithms remains small. Other approaches were based on greedy constructive algorithms (Leighton, 1979; Culberson & Luo, 1996). These algorithms color the vertices of a graph in a sequential manner, adopting a chosen mechanism for selecting the next vertex to color and the color to use. The resulting algorithms are very fast, but not particularly efficient. Metaheuristics methods such as Tabu Search (Hertz & de Werra, 1987), Simulated Annealing (Johnsen, Aragon, McGeoch, & Schevon, 1991), Evolutionary Algorithms (Galinier & Hao, 1999), Minimal-state processing search algorithm (Funabiki & Higashino, 2000), AMACOL (Galinier, Hertz, & Zufferey, 2008), Variable Search Space (Hertz, Plumettaz, & Zufferey, 2008) represent the last category of techniques widely used to solve the graph coloring problem. A recent survey for graph coloring algorithms is provided in (Enrico & Paolo, 2010).

#### 2.2 SAT-Encoded Graph Coloring Approaches

The GCP has been extensively studied due to its simplicity and applicability. The simplicity of the problem coupled with its intractability makes it an ideal platform for exploring new algorithmic techniques. This has led to the development of stochastic local search algorithms for solving the graph coloring problem (Malaguti & Toth, 2010; Johnson, Mehrotra, & Trick, 2008; Chiarandini, Dumitrescu, & Sttzle, 2003; Galinier & Hertz, 2006). When transforming the GCP into SAT (Prestwich, 2004), a decision variant is encoded for which the goal is to find a feasible coloring for a given number of colors. This section aims at presenting a survey of methods used to solve the graph coloring problem via propositional satisfiability.

Due to their combinatorial explosive nature, large and complex SAT-encoded graph coloring problems are hard to solve using systematic algorithms. One way to overcome
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