Supercritical Pitchfork Bifurcation in Implicit Regression Modeling

Stan Lipovetsky, GfK Custom Research North America, USA

ABSTRACT

Chaotic systems have been widely studied for description and explanation of various observed phenomena. The problem of statistical modeling for messy data can be attempted using the so called Supercritical Pitchfork Bifurcation (SPB) approach. This work considers the possibility of applying SPB technique to regression modeling of the implicit functions. Theoretical and practical advantages of SPB regression are discussed with an example from marketing research data on advertising in the car industry. Results are very promising, which can help in modeling, analysis, interpretation, and lead to understanding of the real world data.

Keywords: Chaotic Systems, Implicit Regression, Market Mix Models, Observations, Supercritical Pitchfork Bifurcation

INTRODUCTION

Chaos theory originated a century ago from the works by H. Poincare and A. Lyapunov on the stability of the differential equations solutions, and nowadays it has flourished in a large span of papers on pure and applied mathematics. Numerous applications of the chaos theory can be found in various branches of natural and social sciences (for several examples, see Lorenz, 1993; Strogatz, 2000). Studying on chaotic dynamics of nonlinear deterministic equations shows that many statistical random processes can be described as deterministic systems with non-linear effects (Lele, 1994; Chan & Tong, 2001).

This paper considers a possibility of applying some results from chaos theory to finding a good approximation for messy data characterized by a wide range of the response variable values at each point of the predictors’ values. Several approaches can be used for modeling with such a data, for instance, regressions with dummy variables for changing the regression behavior, regressions for implicit functions with non-unique values of the dependent variable, latent class regressions, regressions by segmented data, and quantile models (see Lipovetsky, 1980, 2007a; Lipovetsky & Conklin, 2005a, 2005b; Koenker, 2005). The current work considers regression modeling based on the bifurcation behavior known in chaotic systems.

DOI: 10.4018/jalr.2010100101

Copyright © 2010, IGI Global. Copying or distributing in print or electronic forms without written permission of IGI Global is prohibited.
Bifurcation is a sudden change in the behavior of a function due to a small variation in the parameters of a system, when the number of solutions and their structure can change abruptly if a parameter passes some critical threshold (Puu, 2000). The current paper considers the so-called Supercritical Pitchfork Bifurcation (SPB) model. This model can be seen as an extension of various sigmoid functions widely used in applied regression modeling. In applications, it describes a simple behavior which at some point transforms into split-path curves. Some illustrations of the SPB diagram are shown in Figure 1 and Figure 2.

Tri-critical points are often related to phase transitions in physics where they correspond to pressure-temperature diagrams of gas, liquid, and solid states, or transitions modeled in social dynamics. In the last dozen years, SPB models have been successfully applied in biology for description of ant colonies complicated behavior (see Bonnabeau, Dorigo, & Theraulaz, 1999; Deneubourg et al., 1989, 1990; Nicolis & Deneubourg, 1999; Camazine et al., 2001; Sumpter & Pratt, 2003; Garnier, Gautrais, & Theraulaz, 2007). Similar models have been tried in fuzzy decision making and design of autonomous robots problems (Sousa & Kaymak, 2002; Rozin & Margaliot, 2007).

This work explores a possibility of applying SPB techniques to regression modeling in such cases when it is difficult to construct a regular regression because of a non-unique dependency of the theoretical variable \( y \) on the independent variable \( x \). On the example of market mix modeling for advertising in the car industry, the theoretical and practical advantages of SPB regression are discussed. The results show that the SPB model helps in analysis, interpretation, and better understanding of the real world data.

**SUPERCRITICAL PITCHFORK BIFURCATION MODELING**

Let us describe briefly some relations of the supercritical pitchfork bifurcation. The SPB differential equation in a normal form can be presented as:

\[
\frac{dy}{dt} = xy - y^3
\]

where \( x \) defines the bifurcation parameter. The steady-state solutions are attained when the derivative equals zero, \( y(x - y^2) = 0 \), with the set of solutions:

![Figure 1. Supercritical pitchfork bifurcation: solutions (2)](image)
What Have Computational Models Ever Done for Us?: A Case Study in Classical Conditioning
[www.igi-global.com/article/what-have-computational-models-ever-done-for-us/103852?camid=4v1a](www.igi-global.com/article/what-have-computational-models-ever-done-for-us/103852?camid=4v1a)

Integrating Various Data Sources for Improved Quality in Reverse Engineering of Gene Regulatory Networks
[www.igi-global.com/chapter/integrating-various-data-sources-improved/38248?camid=4v1a](www.igi-global.com/chapter/integrating-various-data-sources-improved/38248?camid=4v1a)