Chapter IV
A Gentle Introduction to Fuzzy Logic and Its Applications to Intelligent Agents Design

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ABSTRACT

The purpose of this chapter is to present the key properties of fuzzy logic and adaptive nets and demonstrate how to use these, separately and in combination, to design intelligent systems. The first section introduces the concept of fuzzy sets and their basic operations. The $t$ and $s$ norms are used to define a variety of possible intersections and unions. The next section shows two ways to estimate membership functions, polling experts, and using data to optimize parameters. Section three shows how any function can be extended to arguments that are fuzzy sets. Section four introduces fuzzy relations, fuzzy reasoning, and shows the first steps to be taken to design an intelligent system. The Mamdami model is defined in this section. Reinforcement-driven agents are discussed in section five. Sections six and seven establish the basic properties of adaptive nets and use these to define the Sugeno model. Finally, the last section discusses neuro-fuzzy systems in general. The solution to the inverted pendulum problem is given by use of fuzzy systems and also by the use of adaptive nets. The ANFIS and CANFIS architectures are defined.
BASIC CONCEPTS

Fuzzy Sets vs. Standard Sets

A standard set is a collection of objects. Underlining this concept is a specified universal set \( X \). Thus, when a reference is made to a standard set \( A \), one really refers to the subset \( A \) of \( X \).

Example: Let \( X = \{1, 2, 3, 4, 5, 6, 7\} \) and \( A = \{4\} \). Here \( A \) is considered to be a subset of \( X \) consisting of one element, that is, the integer 4.

Each subset of \( X \) can be identified with a function defined on \( X \), taking the value 1 on elements of the subset and the value 0 on the elements of \( X \) that are not in that subset.

Example: Let \( X = \{1, 2, 3, 4, 5, 6, 7\} \). Let \( A = \{3, 4, 5, 6, 7\} \). This set defines a function \( f \) on \( X \) with \( f(x) = 1 \) for \( x = 3, 4, 5 \) and \( f(x) = 0 \) for \( x = 1, 2, 6, 7 \).

Conversely, if \( g \) is a function on \( X \) with \( g(x) = 1 \) for \( x = 2, 5, 7 \) and \( g(x) = 0 \) for \( x = 1, 3, 4, 6 \), then \( g \) defines the subset \( A = \{2, 5, 7\} \).

A fuzzy subset of \( X \) generalizes the concept of standard set.

Definition: By a fuzzy subset of \( X \), we mean a function from \( X \) into the interval \([0, 1]\).

Example: Let \( X = \{1, 2, 3, 4, 5, 6, 7\} \) and define a function \( g \) from \( X \) into \([0, 1]\) as follows:

\[
g(1) = 0, \ g(2) = .4, \ g(3) = .8, \ g(4) = 1, \ g(5) = .8, \ g(6) = .4, \ g(7) = 0
\]

Then, \( g \) defines a fuzzy subset of \( X \). If we look back to the definition of a standard set, \( f(x) = 1 \) means \( x \) belongs to the (standard) subset \( A \) and \( f(x) = 0 \) means \( x \) does not belong to \( A \). A similar interpretation holds for fuzzy subsets. In this example, \( g(6) = .4 \) means that on a scale of 0 to 1, 6 belongs .4 to the fuzzy subset defined by \( g \).

The statement \( g(1) = 0 \) means that 1 does not belong at all to that subset and \( g(4) = 1 \) means that 4 belongs totally to that subset. The graph of \( g \) is shown on Figure 1.

The interpretation of \( g \) could be “The (fuzzy) set of numbers around 4.” The integer 4 belongs totally to “Numbers around 4.” The integers 1 and 7 are too removed “Numbers around 4,” so they don’t belong at all. The integers 2, 3, 5, 6 belong somewhat, 3 and 5 more than 2 and 6. The function \( g \) is referred to as “the membership function” of the set “Numbers around 4.” The membership function of a standard set takes values in \([0, 1]\); the membership function of a fuzzy set is allowed to take values in \([0, 1]\). Thus standard sets constitute a special case of fuzzy sets.

The standard notation for the membership function of a fuzzy set \( A \) is \( \mu_A \). Thus, in our previous example:

\[
\mu_{\text{Numbers around } 4}(x) = g(x), \quad x = 1, 2, 3, 4, 5, 6, 7.
\]

Another standard notation is to express the set “Numbers around 4” by \(.4/2 + .8/3 + 1/4 + .8/5 + .4/6\). The fact that 1 and 7 are missing from this expression indicates that the membership function on 1 and 7 is 0. In general:

\[
A = \sum_{i=1}^{n} \alpha_i/x_i
\]

indicates

\[
\mu_A(x) = \sup_{x_i=x} \alpha_i.
\]

Thus, \( A = .8/x_1 + .4/x_2 + .2/x_1 + .9/x_2 \) indicates \( \mu_A(x_1) = .8 \) and \( \mu_A(x_2) = .9 \).

The notation:

\[
A = \int_{x \in X} \mu_A(x)/x
\]

where \( \mu \) is the membership function of \( A \) signifies that \( A \) is a fuzzy subset of \( X \) whose membership function is \( \mu \).