Chapter 4
Bipolar Quantum Lattice and Dynamic Triangular Norms

ABSTRACT
Bipolar quantum lattice (BQL) and dynamic triangular norms (t-norms) are presented in this chapter. BQLs are defined as special types of bipolar partially ordered sets or posets. It is shown that bipolar quantum entanglement is definable on BQLs. With the addition of fuzziness, BDL is extended to a bipolar dynamic fuzzy logic (BDFL). The essential part of BDFL consists of bipolar dynamic triangular norms (t-norms) and their co-norms which extend their truth-based counterparts from a static unipolar fuzzy lattice to a bipolar dynamic quantum lattice. BDFL has the advantage in dealing with uncertainties in bipolar dynamic environments. With bipolar quantum lattices (crisp or fuzzy), the concepts of bipolar symmetry and quasi-symmetry are defined which form a basis toward a logically complete quantum theory. The concepts of strict bipolarity, linearity, and integrity of BQLs are introduced. A recovery theorem is presented for the depolarization of any strict BQL to Boolean logic. The recovery theorem reinforces the computability of BDL or BDFL.

INTRODUCTION
The concept lattice is central in logic and set theory. A lattice is a partially ordered set (poset) in which any two elements have a least upper bound or supremum (\(\lor\)) and a greatest lower bound or infimum (\(\land\)). Boolean logic (Boole, 1854) is defined on the bivalent lattice \(\{0,1\}\); Zadeh’s fuzzy logic (Zadeh, 1965) is defined on the fuzzy or real-valued lattice \([0,1]\); YinYang bipolar dynamic logic (BDL) is defined on the YinYang bipolar lattice \([-1,0] \times \{0,1\}\) (Ch. 3).

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Bipolar lattice introduces equilibrium or non-equilibrium-based physical syntax and semantics into lattice theory. Such physical syntax and semantics lead to bipolar quantum entanglement as defined in bipolar universal modus ponens (BUMP) (Ch. 3). In this chapter we reference bipolar lattice as bipolar quantum lattice (BQL) due to its salient feature of bipolar quantum entanglement. This should be distinguished from lattice quantum chromodynamics (lattice QCD) in physics, which is a theory of quarks and gluons formulated on a spacetime lattice.

The properties of BQLs are analyzed in this chapter. Notably, the quantum nature of BQLs is distinguished from truth-based unipolar lattice. Energy equilibrium, balance, and symmetry are formally defined and axiomatically formulated.

Although lattice-ordered triangular norms or t-norms and t-conorms are the fundamental operators in probability, logic, and fuzzy set theory, which play essential roles in computational intelligence, artificial intelligence (AI), cognitive informatics, and decision making, they have a number of truth-based fundamental limitations:

1. The static truth-based nature of classical mathematical abstraction in set theory leads to the well-known closed-world assumption in logical computation that forms a bottleneck in automated reasoning specially in equilibrium-based open-world open-ended dynamic reasoning.
2. Without bipolarity any truth-based logical value (crisp or fuzzy) can’t directly carry bipolar equilibrium-based holistic physical semantics. Therefore, truth-based norms are too “logical” for reasoning on the “illogical” but nevertheless natural or physical aspects such as bipolar interaction, oscillation, disorders, and quantum entanglement (see Ch. 3, Figure 1). This limitation leads to the LAFIP or LAFIB paradox (Zhang, 2009a) (see Ch. 1).

The above two fundamental limitations prevent any unipolar truth-based operator from being directly used for holistic knowledge representation and computation in a world of nonlinear bipolar dynamic equilibria especially for bipolar quantum entanglement. To overcome the limitations, equilibrium-based YinYang bipolar quantum lattices and lattice-ordered bipolar dynamic norms are proposed in this chapter that generalize the classical logical connectives $\land$ and $\lor$ from $\{0,1\}$ and $[0,1]$ to $B_1 = \{-1,0\} \times \{0,1\}$, $B_{\uparrow} = [-1.0] \times [0,1]$, and any other BQLs, respectively. Thus, we have the basic classification of lattices and sets:

1. A classical (unipolar) crisp set $X$ in a universe $U$ is defined in the form of its characteristic function $\mu_X : U \rightarrow \{0,1\}$ which yields the value 1 for elements belonging to the set $X$ and 0 for elements excluded from the set $X$. Evidently, classical set theory is based on Aristotle’s universe of truth objects defined in the unipolar bivalent lattice $\{0,1\}$, where the concept of element in a set is claimed self-evident without the need of proof.
2. A classical (unipolar) fuzzy set (Zadeh, 1965) $X$ in a universe $U$ is defined in the form of its characteristic function $\mu_X : U \rightarrow [0,1]$. The fuzzification preserves the unipolar truth-based property of classical set theory with the addition of an infinite number of levels of membership degrees which can be defuzzified to 0 or 1.
3. A YinYang bipolar crisp set $X$ (Ch. 3) in a universe $U$ of bipolar equilibria is defined in the form of its characteristic function $\mu_X : U \Rightarrow B_1$, where $B_1 = \{-1.0\} \times \{0,1\} = \{(-1,0),(0,0),(0,1),(-1,1)\}$ is a bounded complemented crisp BQL.