Existence of Positive Solutions for Generalized p-Laplacian BVPs

Wei-Cheng Lian, National Kaohsiung Marine University, Taiwan
Fu-Hsiang Wong, National Taipei University of Education, Taiwan
Jen-Chieh Lo, Tamkang University, Taiwan
Cheh-Chih Yeh, Lunghwa University of Science and Technology, Taiwan

ABSTRACT

Using Kransnoskii’s fixed point theorem, the authors obtain the existence of multiple solutions of the following boundary value problem

\[
\begin{align*}
(E) & \left( \varphi_p \left( u^{(n-1)}(t) \right) \right)' + f(t, u(t), \ldots, u^{(n-2)}(t)) = 0, \quad t \in (0,1), \\
(BC) & \left\{ \\
& u^{(i)}(0) = 0, \quad 0 \leq i \leq n-3, \\
& u^{(n-2)}(0) - B_0 \left( u^{(n-1)}(\xi) \right) = 0, \\
& u^{(n-2)}(1) + B_1 \left( u^{(n-1)}(\eta) \right) = 0,
\right.
\end{align*}
\]

where \( 0 < \xi < \eta < 1 \) are given. The authors examine and discuss these solutions.

Keywords: Boundary Value Problem, Existence, Kransnoskii’s Fixed Point Theorem, Multiple Solution, p-Laplacian Operator

1. INTRODUCTION

In this paper, we concern with the existence of multiple solutions for higher order boundary value problem where are given. The authors examine and discuss these solutions (see Box 1).

where \( n \geq 3 \) is a positive integer, \( 0 < \xi < \eta < 1 \) are given and \( \varphi_p(s) \) is the p-Laplacian operator, that is, \( \varphi_p(s) = \left| s \right|^{p-2}s \) for \( p > 1 \). Clearly, \( \varphi_p \) is invertible with inverse \( \varphi_q(s) = \varphi_p^{-1}(s) \). Here \( \frac{1}{p} + \frac{1}{q} = 1 \).

DOI: 10.4018/jalr.2011010105
In recent years, the existence of positive solutions for nonlinear boundary value problems with p-Laplacian operator received wide attention. As we know, two point boundary value problems are used to describe a number of physical, biological and chemical phenomena. Recently, some authors have obtained some existence results of positive solutions of multi-points boundary value problems for second order ordinary differential equations (Wang & Ge, 2007; Yu, Wong, Yeh, & Lin, 2007; Zhao, Wang, & Ge, 2007; Zhou, & Su, 2007). In this paper, we establish the existence of positive solutions of general multi-points boundary value problem (BVP) and related results (Bai, Gui, & Ge, 2004; Guo & Lakshmikantham, 1988; Guo, Lakshmikantham, & Liu, 1996; He & Ge, 2004; Lian & Wong, 2000; Liu, 2002; Ma, 1999; Ma & Cataneda, 2001; Sun, Ge, & Zhao, 2007; Wang, 1997).

In order to abbreviate our discussion, throughout this paper, we assume \( H_1 \) is a Banach space with norm \( \| u \| = \max_{t \in [0,1]} |u(t)| \). And let:

\[
K = \left\{ u \in B : u^{(n-2)}(t) \geq 0 \text{ is a concave function, } t \in [0,1] \right\}.
\]

Obviously, \( K \) is a cone in \( B \).

In order to discuss our results, we need the following some lemmas:

**Lemma 2.0**

Assume that \( E \) is a Banach space and \( P \subset E \) is a cone in \( E \); \( \Omega_1, \Omega_2 \) are open subsets of \( E \), and \( 0 \in \overline{\Omega_1} \subset \Omega_2 \). Furthermore, let \( F : P \cap (\overline{\Omega_2} \setminus \Omega_1) \rightarrow P \) be a completely continuous operator satisfying one of the following conditions:

(i) \( \| Fx \| \leq \| x \| , \forall x \in P \cap \partial \Omega_1 ; \| Fx \| \geq \| x \| , \forall x \in P \cap \partial \Omega_2 ; \)

(ii) \( \| Fx \| \leq \| x \| , \forall x \in P \cap \partial \Omega_1 ; \| Fx \| \geq \| x \| , \forall x \in P \cap \partial \Omega_2 ; \)

In order to abbreviate our discussion, throughout this paper, we assume

\[
\big( H_1 \big) f \in C \left( \left[ 0,1 \right] \times \left[ 0, +\infty \right]^{n-1}, \left[ 0, +\infty \right] \right);
\]

\[
\big( H_2 \big) B_0(s), B_1(s) \text{ are both nondecreasing continuous and odd functions defined on } (-\infty, +\infty) \text{ and at least one of them satisfies the condition that there exists } b \geq 0 \text{ such that } 0 \leq B_i(s) \leq bs \text{ for all } s \geq 0, i = 1, 2.
\]

2. PRELIMINARIES AND LEMMAS

Let:

\[
B = \left\{ u \in C^{(n-2)}[0,1] : u^{(i)} = 0, 0 \leq i \leq n - 3 \right\}.
\]

Then, \( B \) is a Banach space with norm \( \| u \| = \max_{t \in [0,1]} |u^{(n-2)}(t)| \). And let:

\[
K = \left\{ u \in B : u^{(n-2)}(t) \geq 0 \text{ is a concave function, } t \in [0,1] \right\}.
\]

Copyright © 2011, IGI Global. Copying or distributing in print or electronic forms without written permission of IGI Global is prohibited.
Dealing with Interaction for Complex Systems Modelling and Prediction
Walter Quattrociocchi, Daniela Latorre, Elena Lodi and Mirco Nanni (2010).
*International Journal of Artificial Life Research* (pp. 1-11).
http://www.igi-global.com/article/dealing-interaction-complex-systems-modelling/38930?camid=4v1a

Optimized Energy Aware VM Provisioning in Green Cloud Based on Cuckoo Search with Levy Flight
http://www.igi-global.com/chapter/optimized-energy-aware-vm-provisioning-in-green-cloud-based-on-cuckoo-search-with-levy-flight/153824?camid=4v1a