On Possibilistic and Probabilistic Information Fusion

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ABSTRACT

This article discusses the basic features of information provided in terms of possibilistic uncertainty. It points out the entailment principle, a tool that allows one to infer less specific from a given piece of information. The problem of fusing multiple pieces of possibilistic information is and the basic features of probabilistic information are described. The authors detail a procedure for transforming information between possibilistic and probabilistic representations, and using this to form the basis for a technique for fusing multiple pieces of uncertain information, some of which is possibilistic and some probabilistic. A procedure is provided for addressing the problems that arise when the information to be fused has some conflicts.

Keywords: Linguistic Information, Multi-Source Information Fusion, Possibilistic and Probabilistic Representations, Possibilistic Uncertainty, Possibility-Probability Transformation

INTRODUCTION

Information used in decision making generally comes from multiple sources. We are interested in the problem of multi-source information fusion in the case when the information provided has some uncertainty. Two important types of sources of information are electro-mechanical sensors and human observers/experts; this is particularly the case in security environments. We note that sensor-provided information generally has a probabilistic type of uncertainty. Human-observer-provided information, which is generally linguistic in nature, typically introduces a possibilistic type of uncertainty. Here we are faced with a problem in which we must fuse information with different modes of uncertainty. Here we shall discuss an approach to attaining this capability based on the use of probability-possibility transformation. We first discuss the basic features of information provided in terms of possibilistic uncertainty. We point out the entailment principle, a tool that allows one to infer less specific information from a given piece of information. We also discuss the problem of fusing multiple pieces of possibilistic information. We describe a procedure for transforming information between possibilistic and probabilistic representations. We use this to form the basis for a technique for fusing multiple pieces of uncertain information, some of which is possibilistic and some probabilistic. We provide a normalization procedure for addressing the problems that arise when the information to be fused has some conflict.

The most well-known uncertainty representation is the probabilistic model. Assume \( V \) is some variable taking its value in the space

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We recall that for any subset $A$ of $X$ the probability that $V$ lies in $A$ is $P_{rob}(A) = \sum_{i: x_i \in A} p_i$. In the probabilistic framework the normalization property that $\text{Prob}(X) = 1$ requires that $\sum_i p_i = 1$. Probabilistic uncertainty is essentially based on ratio scale information. That is, if we know the ratio $\frac{p_i}{p_j}$ then we can completely determine the probability distribution. Within the framework of probability theory the Shannon entropy is used to quantify the amount of uncertainty associated with a probability distribution $P$

$$H(p) = -\sum_{i=1}^{n} p_i \ln(p_i).$$

### Possibilistic Uncertainty

Assume $V$ is some variable taking its value in the space $X$. Formally under a possibilistic uncertainty model we associate with each $x_i \in X$ a value $\pi_i \in [0, 1]$ indicating the possibility that $x_i$ is the value of $V$. We denote the collection of these as the probability distribution $P$. For any subset $A$ of $X$ the probability that $V$ lies in $A$ is $\text{Poss}(A) = \max_{x_i \in A} \pi_i$. In the possibilistic framework the normalization property that $\text{Poss}(X) = 1$ requires that $\max_{i} \pi_i = 1$. The implication here is that at least one of the $x_i$ has possibility one. We also note from our definition of $\text{Poss}(A)$ that if $B \subseteq C$ then $\text{Poss}(B) \leq \text{Poss}(C)$. Furthermore we have $\text{Poss}(\emptyset) = 0$.

Possibilistic uncertainty often arises from linguistically expressed information via a fuzzy set. Again assume $V$ is some variable whose domain is the set $X$. A proposition or datum associated with this variable is a statement of the form $V$ is $\text{Value}$. Here $\text{Value}$ is some constraint on the possible values of the variable (Zadeh, 2005). For example if $V$ is John’s age then $\text{Value}$ could be “young” or “old”, etc. Using the framework of computing with words introduced by Zadeh (1996, 1999) we can express the meaning of $\text{Value}$ in terms of a fuzzy subset $A$ of $X$. In the case of John’s age we would define “young” as a fuzzy over ages. Using the fuzzy subset $A$ we can induce a possibility distribution $\Pi$ to express our knowledge of the value $V$ by assigning $\pi_i = A(x_i)$. Thus the possibility that $V = x_i$ is the membership grade of $x_i$ in $A$.

In passing we want to make one comment regarding the assignment statement $V$ is $\text{Value}$. In point of fact what we have denoted as the variable $V$ can be more precisely viewed as consisting of an attribute and an object. Thus a variable is of the form $\text{Attribute(object)}$. For example in the statement John is young, the more precise statement is age of John is young. Here we have a datum triple, attribute – object – value. Nevertheless in the following we shall continue to use the term variable.

Returning to our possibility distribution $\Pi$ on $V$. We indicated that for any subset $A$ we have $\text{Poss}(A) = \max_{x_i \in A} \pi_i$. We recall that for any subset $A$ we can associate a function $A: X \rightarrow [0, 1]$ such that $A(x_i) = 1$ if $x_i \in A$ and $A(x_i) = 0$ if $x_i \notin A$. Using this we can express $\text{Poss}(A) = \max_{x_i \in A} [A(x_i) \land \pi_i]$. This formulation allows us to calculate the possibility of fuzzy sets when $A(x_i) \in [0, 1]$ instead of $\{0, 1\}$.

Within the framework of possibility theory the concept of specificity (Yager, 1998; Klir, 2006) plays a role analogous to the role of entropy in probability theory, it measures the uncertainty associated with a possibility distribution. Let $\text{ind}(j)$ be the index of the value in $X$ with the $j^{th}$ largest possibility. Using this we define the specificity of the distribution $\Pi$:

$$\text{Sp} (\Pi) = \pi_{\text{ind}(1)} - \frac{1}{n-1} \sum_{j=2}^{n} \pi_{\text{ind}(j)}.$$

It is the difference between the largest possibility and the average of the others. Since
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