Chapter 16
Symbolic Equation for the Instantaneous Amount of Substance in Linear Compartmental Systems:
Software Furnishing the Coefficients Involved in it

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ABSTRACT

The symbolic time course equations corresponding to a general model of a linear compartmental system, closed or open, with or without traps and with zero input are presented in this chapter. From here, the steady state equations are obtained easily from the transient phase equations by setting the time towards infinite. Special attention is given to the open systems, for which an exhaustive kinetic analysis has been developed to obtain important properties. Besides, the results are particularized to open systems without traps. The software COEFICOM, easy to use and with a user-friendly format of the input of data and the output of results, allows the user to obtain the symbolic expressions of the coefficients involved in the general symbolic equation and all the information necessary to derive the symbolic time course equations for closed or open systems as well as for the derivation of the mean residence times.

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INTRODUCTION

In the analysis of any linear compartmental system consisting of compartments $X_1, X_2, ..., X_n$, there are two problems that must be solved: (1) The forward problem, i.e. to ascertain how the system behaves kinetically for given inputs, assuming connectivity between the compartments and that the values of the non-null fractional transfer coefficients, $K_{ij}$ ($i,j = 1, 2, ..., n; i \neq j$), between compartments $X_i$ and $X_j$ are known, and (2) The inverse problem, i.e., to determine the structure connectivity of the system and to estimate the values of the fractional transfer coefficients (Anderson, 1983; Jacquez, 1996; Jacquez, 2002). Some illustrative contributions on the forward problem are those in references (Anderson, 1983; Chou, 1990; Garcia-Meseguer et al., 2001; Hearon, 1963; Lal & Anderson, 1990; Rescigno, 1956; Rescigno, 1999; Varon, Garcia-Meseguer, & Havsteen, 1995b; Varon, Garcia-Meseguer, Garcia-Canovas, & Havsteen, 1995a). A recent contribution on this inverse problem is that of Juillet et al (2009).

The solution of the forward problem, together with the specific inputs of the substance made, leads to knowledge of the kinetic behavior of the compartmental system under study, i.e. the time variation of the amount of substance in each compartment.

The forward problem requires choosing both a model of the connectivity structure of the compartmental system and a mathematical model to acquire the kinetic behavior of the system. Mathematical model may be the residence time concept or the corresponding set of differential equations (Weiss, 1992). The first mathematical model gives time-independent kinetic parameters such as exit and transit times, MRT, occupancy, turnover time and half-life (Anderson, 1983; Jacquez, 1985; Jacquez, 1999). The second mathematical model furnishes the time course of the amount of substance in any compartment of the system after a specific input in one or more compartments is made (Garcia-Meseguer et al., 2001; Jacquez, 2002; Rescigno, 1956; Rescigno, 1999; Rescigno, 2004; Varon et al., 1995a). From the results obtained with the second mathematical model, the parameters provided by the first method can also be obtained (Anderson, 1983; García-Meseguer et al., 2003; Jacquez, 1985; Jacquez, 1999; Varon, García-Meseguer, Valero, García-Moreno, & García-Canovas, 1995). In this section the second mathematical model mentioned above is used.

The compartmental systems are considered closed if there is no interchange of substance between any compartment of the system and the environment; otherwise they are named open systems. A compartment or a set of interconnected compartment, from which substance cannot leave, is named a simple trap, i.e. material is “trapped” (Anderson, 1983; Jacquez, 1985).

From structural point of view of the compartmental systems and according to Rescigno (1956) one compartment $X_i$ is precursor of another, $X_j$, if the variation in the concentration in the first compartment influences the concentration in the second one. Then $X_j$ is called successor $X_i$. Naturally, a compartment is a precursor and a successor of itself.

In this way, one class is formed by all compartments of the system, such that any of them is a precursor of all the others, and each compartment of a system belongs to one and only to one class (Galvez & Varon, 1981). Therefore, a class is a component of the strongly connected system (Jacquez, 1985). A class of the system which neither transfer material to any other class of the system nor to the environment (in open systems) is designated a final class. The concept of the final class coincides with the definition of simple trap given by Jacquez (1985).

In closed systems without traps the whole system can be considered as a trap and the only existing class is a final class. Hence in systems, which are not strongly connected (with traps), more than one class exits. Next, the study of the