Chapter 10
Developing Teachers’ Mathematical Knowledge for Teaching through Online Collaboration

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ABSTRACT
In this chapter, we discuss our perspective on mathematical knowledge for teaching and present an emerging instructional model for supporting the development of that knowledge through an instructor facilitated online environment. We focus on the ways in which online collaboration can support teachers as they wrestle with, and ultimately make sense of, the mathematical structures that underlie a variety of the school mathematics curriculum. Three cases are provided to highlight the potential that online collaboration hold for supporting teachers as they collaborate about specific mathematical ideas and reflect on those collaborations. We believe that these cases can also serve as a starting point for further conversations about supporting online mathematical collaboration and online mathematics teacher development.

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INTRODUCTION

The importance of teachers' mathematical knowledge has been well documented in the literature (Ball, 1993; Bransford, Brown, & Cocking, 2000; Ma, 1999; Shulman, 1986) and increasing teachers’ mathematical knowledge continues to be a major focus in both education research and policy (Greenberg & Walsh, 2008; National Mathematics Advisory Panel, 2008). In this chapter, we discuss our perspective on mathematical knowledge for teaching and present an emerging instructional model for supporting the development of that knowledge through an instructor facilitated online environment. Because of the scarcity of models for supporting the development mathematical knowledge for teaching for secondary mathematics (face-to-face or online), the primary focus of this chapter will be on detailing three “case studies” of teachers engagement and collaboration about one big mathematical idea that underlies a large portion of the school mathematics curriculum: similarity, equality, and congruence. These cases will highlight the potential online collaboration holds for supporting teachers as they collaboratively engage with these big mathematical ideas and reflect on those collaborations. We believe that these cases can also serve as a starting point for further conversations about supporting online mathematical collaboration.

THEORETICAL BACKGROUND

Mathematical Knowledge for Teaching

Ball and her colleagues (Ball, 1993, 2007; Ball, Hill, & Bass, 2005; Ball & McDiarmid, 1990) have focused on understanding the special ways one must know mathematical procedures and representations to interact productively with students in the context of teaching. Their pioneering work has succeeded in identifying a statistical relationship between this mathematical knowledge for teaching (MKT) and student achievement (Ball, et al., 2005; Hill, Rowan, & Ball, 2005). We extend this work by focusing not only on particular mathematical understandings but also the conceptual structures within which those particular understandings lie. Our reason for this focus is pragmatic:

*If a teacher’s conceptual structures comprise disconnected facts and procedures, their instruction is likely to focus on disconnected facts and procedures. In contrast, if a teacher’s conceptual structures comprise a web of mathematical ideas and compatible ways of thinking, it will at least be possible that she attempts to develop these same conceptual structures in her students (Thompson, Carlson, & Silverman, 2007)*

Rather than focusing on identifying the mathematical reasoning, insight, understanding and skill needed in teaching mathematics, we focus on the mathematical understandings “that carry through an instructional sequence, that are foundational for learning other ideas, and that play into a network of ideas that does significant work in students’ reasoning” (Thompson, 2008). We refer to these understandings as coherent understandings: powerful, generative “big ideas” from which an understanding of a body of mathematical ideas and its relation to other bodies can emerge.

It is important to note that coherence is not a characteristic of one’s understanding of a particular mathematical idea, for coherence in curricula or students’ understandings depends on the way in which they fit together (Thompson, 2008). This notion of coherence is a challenge to traditional mathematics teacher education efforts that seek to support teachers in “gain[ing] the ability to do the mathematics … and understand[ing] the underlying concepts so they will be able to assist their students, in turn, to gain a deep understanding of mathematics” (Musser, Burger, & Peterson, 2008). When a focus is on coherence, the emphasis is not just on doing and learning “the mathematics,”