Two-Facility Location Problem with Infinite Retrial Queue

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ABSTRACT

This paper analyzes a two-facility location problem under demand uncertainty. The maximum server for the ith facility is $M_i (i = 1, 2)$. It is assumed that primary service demand arrivals for the ith facility follow a Poisson process. Each customer chooses one of the facilities with a probability which depends on his or her distance to each facility. The service times are assumed to be exponential and there is no vacation or failure in the system. Both facilities are assumed to be substitutable which means that if a facility has no free server, the other facility is used to fulfill the demand. When there is no idle server in both facilities, each arriving primary demand goes into an orbit of unlimited size. The orbiting demands retry to get service following an exponential distribution. In this paper, the authors give a stability condition of the demand satisfying process, and then obtain the steady-state distribution by applying matrix geometric method in order to calculation of some key performance indexes. By considering the fixed cost of opening a facility and the steady-state service costs, the best locations for two facilities are derived. The result is illustrated by a numerical example.

Keywords: Discrete Facility Location, Matrix Geometric Method, Quasi-Birth-And-Death Process, Queuing Theory, Retrial Queue

INTRODUCTION

Decision making in facility location problems is costly and difficult to reverse and its impact spans a long time horizon. During the time when designed decisions are in operation, each parameter in problem (such as costs, demands, distances, and service times) may vary widely. Estimated parameter may also be imprecise due to poor measurements or tasks inherent in the modeling process. Recognizing this, developed models for facility location under uncertainty have been subjected matter for many researchers for several decades.

There are some services, such as emergency vehicles, car rental agencies, among others, for which ‘mobile’ servers must travel to the site of the request so as to satisfy the demand, as opposed to ‘immobile’ servers that, in order to obtain service, the customer must travels to the service facility from the place where it is normally located. In some cases, customer cannot
be served due to the fact all available servers in facility are occupied by other customers.

The first probabilistic model that considered queueing theory in facility location problems is the hypercube queueing model discussed by Larson (1974); it analyzes problems of vehicle location-allocation and response district design in emergency response services that operate in the server to customer mode. Quite a lot of models have been presented which are explicitly intended for the facility location problem with congestion. Generally, these models are oriented to emergency systems, in which servers travel to the site of emergency request. Daskin and Stern (1981) considered emergency services with mobile servers and maximized the total number of servers acting as backup, while keeping total coverage by main servers. A spatial queueing model for an emergency vehicle districting and location problem is studied by Geroliminis et al. (2009) where they develop a model for locating emergency vehicles on metropolitan networks considering both spatial and temporal demand characteristics such as the probability that a server is not available when required. A heuristic algorithm is developed by Berman et al. (1985), to locate optimally a single server on congested network. Batta (1989) presented a model which considers the effect of using expected service time dependent queuing disciplines on optimal location of a server.

Facility location with several immobile servers subject to congestion has been studied mostly through formulations that represent each facility as \( M / M / m \) queuing systems (Marianov & Serra, 1998, 2001, 2002). A location of multiple-server congestible facilities for maximizing expected demand is analyzed by Marianov (2003). He formulates a model for locating multi-server congestible facilities which maximizes total expected demand attended over the region and proposes a heuristic for the resulting integer, nonlinear formulation. Recently, Marianov et al. (2008) presented a model which studies the optimal location of multi-server congestible facilities which operate as \( M / E_r / m / N \) queues and analyzes the influence of the service time parameters and the capacity of the facilities on the system performance. Snyder (2006) illustrates both the rich variety of optimization approaches under uncertainty that have appeared in the literature and their application to facility location problems.

In this paper, we address the problem of the lowest cost locating service centers with mobile servers under the presence of demand uncertainty and retrial queue. Retrial queues are very pervasive in most realistic systems. Retrial queueing system is characterized by the feature that the arriving customers who encounter the busy server join a retrial queue called orbit but after some random time returns to the system again. Consequently, in multi-facility systems, repeated attempts from unsatisfied customer’s retrial queue for getting service treat like the primary selection probabilities of first attempt arrivals. Such queueing systems play an important role in the analysis of many telephone systems, call centers, and telecommunication networks. A number of applications of retrial queues both in science and engineering can be seen in Kulkami and Liang (1997).

Diamond and Alfa (1999) structured a method for approximating the stationary distribution and waiting time moments of a \( M / PH / 1 \) retrial queue with phase type inter-retrial times. A retrial queue under Bernoulli vacation schedules is discussed in Ke and (2009) where they consider a batch arrival retrial queue with general retrial times. New descriptors of the customer’s behavior in a \( M / G / 1 \) retrial queue is analyzed by Amador and Artalejo (2009) where they investigate the distribution of the successful retrials, the successful arrivals and the blocked retrials. Moreover, Kumar et al. (2009) analyze a multi-server feedback retrial queueing system with finite waiting position as a quasi-birth-and-death (QBD) process and investigate the necessary and sufficient condition for stability of the system.

To our best knowledge, in facility location literature there is no work about the effect of retrial demand on locating facilities. Based on the frequent unsatisfied demands and customer satisfaction level importance in service facili-
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