Chapter 14

Stochastic Programming in Supply Chain

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ABSTRACT

In today’s global market, Supply Chain Management provides a framework on which decision makers can prepare a production or service network to be more competitive. The huge number of decisions as well as the underlying uncertainty make this system too complex. Stochastic programming as an optimization technique, which incorporates uncertainty in decisions, has been employed in this area. In this chapter, the authors will investigate some applications of stochastic programming in Supply Chain. The authors also explain briefly about stochastic programming.

1. INTRODUCTION

Almost all areas of science and engineering, from health care and medicine to logistic and power distribution systems, contain optimization models. This vast area of applications is, naturally, of interest for numerous researchers to formulate the problems, analyze different aspects of them, and eventually, find the best or at least a good solution for them. What makes a formulation more realistic is to assume their uncertain natures. On the other hand, Mason-Jones and Towill (1998) state that the companies which incorporate uncertainty in their decision making procedure, make more profit. Moreover, they have more chance to produce competitive bottom-line performance. Although there have been already many proposed ways to model uncertain information, solid mathematical and theoretical foundations of stochastic models have proved their preferences to other models. Due to the importance of optimization in uncertain environments, theory and methods of stochastic programming have recently experienced very fast and growing advances.

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On the other hand, in the global market firms have figured out that they no longer will be able to compete individually. That is, the idea of Supply Chain as an integrated system has been growing rapidly. The more efficient the chain is, the more profit would make the whole chain. This is what encouraged managers to spend a fair amount of time and money to have the best possible outcomes of the chain by using appropriate methods of optimization.

These are quite persuasive reasons for us to study some models in Supply Chain, as one of the recent stimulus applications, involving uncertain parameters for which stochastic models are available. In section 2, we explain about stochastic programming, in particular two-stage stochastic linear programming, and some of its properties and approaches to solve it. Then in section 3, some related properties of Supply Chain, such as time horizons and source of uncertainty in Supply Chain, are explained. The basic models and problems of Supply Chain which can be solved with stochastic programming will be described in section 4. Finally, we cover two basic models which have used two specific methods of stochastic programs.

2. TWO-STAGE STOCHASTIC LINEAR PROGRAM

In this section, we will mainly explain methods developed for solving the two-stage stochastic programs. Note that this area of knowledge is too vast to be explained in one chapter. That is, it only suffices for us to have a brief review.

First, we look at the general form of two-stage linear programs. Then, we will explain the three main methods, including L-Shape method, Lagrangian relaxation and Sample Average Approximation, have been proposed for solving the problem.

2.1. Two-Stage Linear Problems

2.1.1. An Illustrative Example

Suppose that we have a transportation network consisting of \( I \) warehouses and \( J \) markets. Each warehouse has a certain amount of good, say \( s_i, i = 1,\ldots,I \) to supply the demand of markets, say \( d_j, j = 1,\ldots,J \). The cost of transporting one unit of the good from warehouse \( i \) to market \( j \) is represented by \( c_{ij} \). Let \( x_{ij} \) show the amount that we decide to transport from warehouse \( i \) to market \( j \). Therefore, the linear program of the described problem would be:

\[
\begin{align*}
\min_Z & \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} x_{ij} \\
\text{subject to} & \sum_{j=1}^{J} x_{ij} \leq s_i, i = 1,\ldots,I, \\
& \sum_{i=1}^{I} x_{ij} \geq d_j, j = 1,\ldots,J, \\
& x_{ij} \geq 0, i = 1,\ldots,I, j = 1,\ldots,J.
\end{align*}
\]

If \( \sum_{i=1}^{I} s_i \geq \sum_{j=1}^{J} d_j \), then the above linear program is always feasible (Winston, 2004). Otherwise, if the total supply is less than the total demand, then there is no feasible assignment. In this case, a dummy variable, say \( y_j \), is defined for each market showing its demand shortage with a large cost coefficient. Therefore, in this case the model looks like as follows:

\[
\begin{align*}
\min_{x,y} Z & = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} x_{ij} + \sum_{j=1}^{J} M y_j \\
\text{subject to} & \sum_{j=1}^{J} x_{ij} \leq s_i, i = 1,\ldots,I, \\
& \sum_{i=1}^{I} x_{ij} + y_j \geq d_j, j = 1,\ldots,J.
\end{align*}
\]