Interval Cut-Set of Generalized Interval-Valued Intuitionistic Fuzzy Sets

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ABSTRACT

In this paper, some different types of interval \((\alpha, \beta)\) cut-set of generalized interval-valued intuitionistic fuzzy sets (GIVIFSs), complement of these \((\alpha, \beta)\) cut-sets are introduced. Some properties of those \((\alpha, \beta)\) cut-set of GIVIFSs are investigated. Also three decomposition theorems of GIVIFSs are obtained based on the different \((\alpha, \beta)\) cut-set of GIVIFSs. These works can also be used in setting up the basic theory of GIVIFSs.

Keywords: Decomposition Theorem, Generalized Interval-Valued Intuitionistic Fuzzy Sets (GIVIFSs), Interval Cut-Set on Generalized Interval-Valued Intuitionistic Fuzzy Sets, Interval-Valued Intuitionistic Fuzzy Sets (IVFSs), Intervals

INTRODUCTION

First the conception of fuzzy subset was given by Zadeh (1965). Latter many authors defined different directions of fuzzy subsets like vague set, rough set, soft set etc.. Turksen (1986) generalized the concept of fuzzy set in terms of interval-valued fuzzy set (IVFS). Several researchers present a number of results using IVFSs. Using these concept of (IVFS) Pal and Shyamal (2006) introduced interval-valued fuzzy matrices and shown several properties of them. Atanassov (1986) and Atanassov and Gargo (1989) introduced the concept of intuitionistic fuzzy sets (IFSs), which is more generalization of fuzzy subsets and as well as IVFSs. Several authors present a number of results using IFSs. By the concept of IFSs, first time Pal (2001) introduced intuitionistic fuzzy determinant. Latter on Pal, Khan, and Shyamal (2002), introduced intuitionistic fuzzy matrices and distance between intuitionistic fuzzy matrices. Recently, Bhowmik and Pal (2008) introduced some results on intuitionistic fuzzy matrices, intuitionistic circulant fuzzy matrices and generalized intuitionistic fuzzy matrices. After the work of Atanassov (1986), again Atanassov and Gargo (1989) introduced the interval-valued intuitionistic fuzzy sets (GIVIFSs). They have shown several properties

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In the theory of fuzzy systems, the cut-sets of fuzzy sets play an important role (Li & Li, 2008) which reveals the relationship between the fuzzy sets and classical sets. Decomposition theorem can be obtained based on the cut-sets. In Yuan (1997), the cut-sets of the fuzzy sets can also be described by neighborhood relation of the fuzzy point and fuzzy set, which has many applications in fuzzy topology and fuzzy algebra.

The organization of this paper is as follows. In the first section, some basic properties of GIVIFS are redefined. Followed by different types of interval cut-sets of GIVIFS are defined and some properties of these cut-sets are given. Then we discuss three decomposition theorems on interval cut-sets of GIVIFSs are gained. Finally, a conclusion of this paper is given.

**PRELIMINARIES**

In this section, the concept of interval arithmetics are recalled. Let $[I]$ be the set of all closed subintervals of the interval $[0,1]$. An interval on $[I]$, say $\alpha$, is a closed subinterval of $[I]$ i.e., $\alpha = [a^-, a^+]$ where $a^-$ and $a^+$ are lower and upper limits of $\alpha$ respectively and satisfy the condition $0 \leq a^- \leq a^+ \leq 1$. For any two interval $\alpha$ and $\beta$ where $\alpha = [a^-, a^+]$ and $\beta = [b^-, b^+]$ then (i) $\alpha = \beta \iff a^- = b^-$, $a^+ = b^+$, (ii) $\alpha \leq \beta \iff a^- \leq b^-$, $a^+ \leq b^+$ and (iii) $\alpha < \beta \iff a^- < b^-$, $a^+ < b^+$ and $\alpha \neq \beta$.

**Definition 1.** An IVIFS $A$ over $X$ (universe of discourse) is an object having the form $A = \{x, M_A(x), N_A(x) \mid x \in X \}$, where $M_A(x) : X \rightarrow [I]$ and $N_A(x) : X \rightarrow [I]$. The intervals $M_A(x)$ and $N_A(x)$ denote the intervals of the degree of membership and degree of non-membership of the element $x$ to the set $A$, where $M_A(x) = [M_{AL}(x), M_{AV}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AV}(x)]$, for all $x \in X$, with the condition $0 \leq M_{AV}(x) + N_{AV}(x) \leq 1$. For simplicity, we denote $A = \{(x, [A^-(x), A^+(x)], [B^-(x), B^+(x)]) \mid x \in X \}$.

Let $\mathbb{F}(X)$ be the set of all GIVIFSs defined on $X$.

**Definition 2.** If the IVIFS $A = \{x, M_A(x), N_A(x) \mid x \in X \}$, satisfying the condition $M_{AV}(x) \land N_{AV}(x) \leq 0.5$ for all $x \in X$ then $A$ is called generalized interval-valued intuitionistic fuzzy set (GIVIFS). The condition $M_{AV}(x) \land N_{AV}(x) \leq 0.5$ is called generalized interval-valued intuitionistic fuzzy condition (GIVIFC). The maximum value of $M_{AV}(x)$ and $N_{AV}(x)$ is 1.0, therefore GIVIFC imply that $0 \leq M_{AV}(x) + N_{AV}(x) \leq 1.5$.

It may be noted that all IVIFS are GIVIFS but the converse is not true.

**Some Operations on GIVIFSs**

In 2009, Bhowmik and Pal defined some relational operations on GIVIFSs. Let $A$ and $B$ be two GIVIFSs on $X$, where

$$A = \{([M_{AL}(x), M_{AV}(x)], [N_{AL}(x), N_{AV}(x)] : x \in X \}$$

and

$$B = \{([M_{BL}(x), M_{BV}(x)], [N_{BL}(x), N_{BV}(x)] : x \in X \}.$$  

Then, $A \leq B \iff M_A(x) = M_B(x)$ and $N_A(x) = N_B(x)$ for all $x \in X$.  

$$A = B \iff M_A(x) = M_B(x) \land N_A(x) = N_B(x)$$  

(1)