Chapter 1

Graph Matching Techniques for Computer Vision

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ABSTRACT

Many computer vision applications require a comparison between two objects, or between an object and a reference model. When the objects or the scenes are represented by graphs, this comparison can be performed using some form of graph matching. The aim of this chapter is to introduce the main graph matching techniques that have been used for computer vision, and to relate each application with the techniques that are most suited to it.

INTRODUCTION

A crucial step in many Computer Vision applications is the comparison between two objects, or between an object and a reference model. For instance, in Object Recognition it is necessary to compare a description of a given object with a description of a larger scene in order to find whether, and where, the object occurs within the scene. In Object Classification, an object needs to be compared with the descriptions of a set of classes that are significant for the particular applications, in order to ascribe the object to one of these classes. In Object Tracking, an object detected in a frame acquired by a camera has to be compared with all the objects present in the previous frame, for reconstructing the trajectories of the objects. Many more examples could be found in virtually all Computer Vision applications.

How the comparison is performed is obviously dependent on the representation adopted for describing the objects of interest, or the scene containing them. In statistical approaches, objects are represented by a vector of global, numerical...
features, and comparison is based typically on a vector norm or distance measure, possibly taking into account probabilistic considerations. Different approaches, the so-called structural ones, represent the objects using simpler primitives (e.g. regions, lines and so on) and their spatial relations, in order to capture and to exploit the information conveyed by the very structure of the objects. In these approaches, object representations are typically given in terms of strings or graphs; in both cases it is not possible to use the well-known vector operations to compare the objects, but a more complex algorithm is needed. In particular, if a Graph-based representation is adopted for the object/scene descriptions, the comparison entails some form of Graph Matching, the determination of a mapping between the nodes/edges of a graph and the nodes/edges of a second, possibly larger, graph. In the literature there are literally hundreds of Graph Matching algorithms, differing with respect to the properties of the desired mapping, the methodology used to find it, the kind of graphs supported, the techniques adopted for keeping the problem solvable in a reasonable time. For instance, there are exact matching algorithms, where the desired mapping must strictly preserve the node adjacency relationships, and inexact matching algorithms, where the structure of the two graphs being compared can be more or less different, and a strict correspondence is not required. In optimal matching algorithms, the goal is to find the global optimum of a measure characterizing the quality of the found mapping, while in approximate matching algorithms global optimality is sacrificed in name of speed. General matching algorithms support all kinds of graphs; specialized algorithms, instead, can be used only with particular classes of graphs (for instance, trees or planar graphs).

Given this large number of algorithms, it is not easy to choose the right one for a particular Computer Vision problem. The aim of this chapter is to provide some guidance for this purpose, introducing the matching techniques that have been most commonly used in Computer Vision, discussing the peculiarities of each problem that make it particularly suitable (or particularly unsuitable) for a specific technique, and finally presenting the recent trends in the literature, such as the growing interest in techniques based on graph kernels.

**GRAPH MATCHING ALGORITHMS**

In this section we will present a short introduction to the most commonly used graph matching techniques. The techniques are grouped on the basis of how the matching problem is formulated, giving rise to the following categories:

- **Exact Graph Matching Algorithms**: Have strict requirements on the preservation of the structure of the graphs across the mapping
- **Inexact Graph Matching Algorithms**: Tolerant about structure differences in the parts of the graphs being matched
- **Graph Embeddings and Graph Kernels**: Techniques to apply algorithms developed for vectorial representations to the graph domain; such techniques are gaining a growing interest in the recent literature
- **Other Matching Techniques**: The structure of the graphs plays only a minor role

**Exact Graph Matching**

Exact graph matching is characterized by the fact that the mapping between the nodes of the two graphs must be edge-preserving in the sense that if two nodes in the first graph are linked by an edge, they are mapped to two nodes in the second graph that are linked by an edge as well.

Conceptually, the simplest form of graph matching is graph isomorphism, where an exact