Chapter 2

Geometric-Edge Random Graph Model for Image Representation

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ABSTRACT

This chapter presents a random graph model for image representation. The first contribution the authors propose is a Geometric-Edge (G-E) Random Graph Model for image representation. The second contribution is that of casting image matching into G-E Random Graph matching by using the random dot product graph based matching algorithm. Experimental results show that the proposed G-E Random Graph model and matching algorithm are effective and robust to structural variations.

INTRODUCTION

Image modeling methods can be divided into two categories, one is spatial domain methods, and the other is transformation domain methods. In this chapter, we focus on spatial domain. In spatial domain, the traditional image models include polygon, skeleton, chain code, run length code, pyramid, and graphs. Recently, more research interests are focused on high level methods, such as graphs. The advantages of the graph model include its capable of describing structural relationships between image units and there exists solid problem solving schemes from mathematics.

BACKGROUND

There exist many graph models to represent images and object shapes. The widely used models include region adjacency graph, MST, Delaunary graph, K-NN graph, Geometric graph, Shock graph and so on. When images are segmented into different regions, adjacency graphs can be used to represent images by assigning each separate region to a graph node. A graph edge is added when the two regions are connected. Image key points from Harris or SIFT like algorithms can be used to generate geometric graphs such as Delaunay graph, K-NN graph, et al.

However, due to the imperfect image segmentation step or key point extraction step, the graphs generated usually have additional and missing nodes, and the edge structure may not be stable. In order to capture the structural variations more effectively, we explore random graph model (Bagdanov, 2003) instead of traditional graph methods.

As a kind of random graph model, complex networking has been explored in image and object representation successfully (Antiqueira, 2005) (Backes, 2010) (Backes, 2009) (Chalumeau, 2006) (Costa, 2005) (Costa, 2004) (Erdős, 1961) (Watts, 1998). The main idea is that we represent images and objects by complex network, then describe the generated complex network by analyzing their topological and dynamic characteristics.

In this chapter, we propose a generalized edge random graph model (Elizabeth, 2010) which is induced by the random geometric graph. We denote this kind of random graph model by Geometric-Edge Random Graph, or G-E Random Graph. The main idea for the G-E Random Graph is that the probability of an edge connecting nodes i and j is determined by random geometric graph model (Penrose, 2003).

To verify the effectiveness of the proposed random graph model, we pursue graph matching based on the G-E Random Graph model. In real world problem domain, since the process of graph generation from raw image data is a task of some fragility due to noise and non-rigid deformation, graph matching is inevitably inexact in nature. There is a considerable literature on the problem of inexact graph matching (Bai, 2004) (Christmas, 1995) (Gold, 1996) (Huet, 1999) (Luo, 2001) (Meyers, 2000) (Scott, 1991) (Shapiro, 1992) (Tang, 2011) (Umeyama, 1988). Broadly speaking, the first category is based on tree search techniques. The second one is based on relaxation and optimization techniques. More recently, spectral methods (Bai, 2004) (Caelli, 2004) (Luo, 2001) (Tang, 2011) are shown effective for graph matching.

Although many inexact graph matching and similarity measurement methods such as spectral method, probability method for graphs with structural variation have been successfully developed, they will usually collapse when the structural variation extends to some level. Contrast to the traditional graph matching algorithms, the proposed G-E Random Graph model has shown its robustness to structural variations and hence achieved higher correct matching rates.

**GEOMETRIC-EDGE (GE) RANDOM GRAPH MODEL**

In this section, we develop the G-E Random Graph model starting from the general geometric graph. As a traditional graph model, geometric graph can be used to extract structure information of an image.

**Geometric Graph**

**Definite 1:** Let $||\cdot||$ be some norm (such as Euclidean norm) on $\mathbb{R}^d$, and let $r$ be some positive parameters. Given finite sets $X, Y \subseteq \mathbb{R}^d$, a geometric graph $G(X; r)$ is an undirected graph with vertex set $X$ and with undirected edges connecting all pairs $\{X, Y\}$ if they satisfy $||Y-X|| \leq r$.

If $||Y-X|| \leq r$, then $X$ and $Y$ are called mutual neighbor. There is an edge connecting $X$ and $Y$. In image matching and recognition, we can use