Chapter 13
Semantic Manipulations and Formal Ontology for Machine Learning Based on Concept Algebra

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ABSTRACT
Towards the formalization of ontological methodologies for dynamic machine learning and semantic analyses, a new form of denotational mathematics known as concept algebra is introduced. Concept Algebra (CA) is a denotational mathematical structure for formal knowledge representation and manipulation in machine learning and cognitive computing. CA provides a rigorous knowledge modeling and processing tool, which extends the informal, static, and application-specific ontological technologies to a formal, dynamic, and general mathematical means. An operational semantics for the calculus of CA is formally elaborated using a set of computational processes in real-time process algebra (RTPA). A case study is presented on how machines, cognitive robots, and software agents may mimic the key ability of human beings to autonomously manipulate knowledge in generic learning using CA. This work demonstrates the expressive power and a wide range of applications of CA for both humans and machines in cognitive computing, semantic computing, machine learning, and computational intelligence.

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1. INTRODUCTION

Concepts are the basic unit of semantic cognition that carries certain meanings in expression, thinking, reasoning, and system modeling (Smith & Medin, 1981; Wille, 1982; Medin & Shoben, 1988; Murphy, 1993; Codin et al., 1995; Zadeh, 1999; Ganter & Wille, 1999; Wilson & Keil, 2001). In denotational mathematics (Wang, 2007a, 2008d), a concept is formally modeled as an abstract and dynamic mathematical structure that encapsulates a coherent hierarchy of attributes, objects, and relations (Wang, 2010b). The formal methodology for manipulating knowledge by concept algebra is developed by Wang (2008c), which provides a generic and formal knowledge manipulation means for dealing with complex knowledge and language structures as well as their algebraic operations.

Knowledge and concepts may be represented by ontology, which is the branch of metaphysics dealing with the nature of being in philosophy (Wilson & Keil, 2001). However, in computing and AI, ontology (Cocchiarella, 1996; GOLD, 2010) is both a method for modeling a domain of knowledge and an entity that represents a part of knowledge in knowledge engineering. Ontological engineering is a method of knowledge engineering. Typical ontological systems are WordNet – a lexical knowledgebase (Miller et al., 1990; Miller, 1995; Vossen, 1998), Dublin Core – an ontology for documents and publishing as standardized in ISO 15836 (ISO, 2011), and GOLD – a general ontology for linguistic description (GOLD, 2010). However, ontological technologies may only represent a set of static knowledge and are highly application-specific and subjective, which may not allow machines to mimic the process of human ontology building. In order to solve his problem, a mathematical model of general concept is formally elicited as an abstract and dynamic mathematical structure that denotes a concept as a triple of sets of attributes, objects, and relations (Wang, 2007c).

Based on the mathematical model of concepts, a formal methodology for manipulating knowledge is developed known as concept algebra (Wang, 2008c, 2010b), which provides a generic and formal knowledge manipulation means for dealing with complex knowledge and natural language structures as well as their algebraic operations. In concept algebra, the formal methodology for visualizing knowledge as a concept network is enabled for knowledge engineering.

It is recognized that new types of problems require new forms of mathematics. The maturity of a discipline is characterized by the maturity of its mathematical means. The family of mathematics may be classified into analytic, numerical, and denotational mathematics (Bender, 2000; Wang, 2008d). Analytic mathematics deals with mathematical entities on \( \mathbb{R} \) (real numbers) with static relations and deterministic functions. Numerical mathematics deals with mathematical entities on \( \mathbb{R} \) or \( \mathbb{B} \) (bits) with discrete and recursively approximate functions. However, the domain of problems in machine learning and cognitive computing are hyper-structures (HΣ) beyond that of pure numbers on \( \mathbb{R} \). Therefore, the requirement for reduction of complex knowledge onto the low-level data objects in conventional computing technologies and their associated analytic mathematical means has greatly constrained the inference and computing ability toward the development of intelligent knowledge processors known as cognitive computers (Wang, 2009c). This observation (Bender, 2000; Wang, 2008d) has triggered the current transdisciplinary investigation into new mathematical structures for machine learning and cognitive computing collectively known as denotational mathematics.

Definition 1: A hype-structure, HΣ, is a type of mathematical entities that is a complex \( n \)-tuple with multiple fields of attributes and constraints, as well as their interrelations, i.e.: