INTRODUCTION

Vibro-impact systems appear in different mechanical problems (modeling of impact dampers, clock mechanisms, immersion of constructions, etc.). All the impact systems are strongly nonlinear. Their properties resemble ones of classical nonlinear systems. Particularly, the chaotic dynamics is possible (Akhmet, 2009; Chin, Ott, Nusse, & Grebogi, 1995; Fredriksson & Nordmark, 1997; Holmes, 1982; Kryzhevich & Pliss, 2005; Nordmark, 1991; Thomson & Ghaflari, 1983; Whiston, 1987).

There are hundreds of publications, devoted to bifurcations, proper to vibro-impact systems. One of these bifurcations, the so-called grazing bifurcation, first described by Nordmark (1991), corresponds to the case, when there is a family of periodic solution, which has a finite number of impacts over the period and this number increases or decreases as the parameter changes. For the bifurcation value of the parameter, the periodic solution has an impact with a zero normal velocity. It was shown that this bifurcation implies a non-smooth behavior of solutions, instability of the periodic solution in the parametric neighborhood of grazing and, in additional assumptions, the chaotic dynamics (Budd, 1995; Chin, Ott, Nusse, & Grebogi, 1995; Fredriksson & Nordmark, 1997; Ivanov, 1996; Nordmark, 1991; Whiston, 1987).

The approaches to find a chaos in impact systems are very different. For example, topological Li-Yorke chaos was studied by Akhmet (2009) and di Bernardo, Budd, Champneys, and Kowalczyk (2007). Stochastic chaos (existence of SRB-measures) was considered by Bunimovich, Pesin, Sinai, and Jacobson (1985) and Chernov and Markarian (2001).

Devaney’s chaos was studied for single degree of freedom systems in the author’s work Kryzhevich (2008). In this paper generalize this result for systems with several degrees of freedom. The main result of this paper is
the method, which allows finding homoclinic points, corresponding to grazing. The main idea of the proof is the nonsmoothness of Perron surfaces in the neighborhood of periodic solution. If these manifolds bend in a “good” way (the corresponding sufficient conditions can be written down explicitly) they can intersect. This implies chaos. We study a motion of a point mass, described by system of second order differential equations of the general form and impact conditions of Newtonian type.

Unlike the similar author’s paper Kryzhevich (2008) here we study the systems with several degrees of freedom. For these systems a near-grazing periodic point is not automatically hyperbolic, so we need to provide additional conditions to have a classical Smale horseshoe.

In order to avoid technical troubles we assume that the delimiter is plain, immobile and slippery. However, there are no obstacles to apply the offered method to the systems with a mobile delimiter (e.g., Holmes, 1982), ones with non-Newtonian model of impacts (Babitsky, 1998; Fredriksson & Nordmark, 1997; Ivanov, 1996; Kozlov & Treshev, 1991) and even to some special cases of strongly nonlinear dynamical systems without any impact conditions.

The paper is organized as follows. First, we consider the mathematical model of vibro-impact systems and then define the grazing family of periodic solution. Afterwards, the near-grazing behavior of solutions is described. Next, the main result of the paper is presented and the near-impact behavior of solutions is studied and estimates of Lyapunov exponents are given. In the next section an analogue of the Smale horseshoe was constructed and an analogue of the Smale-Birkhof theorem is proved. An example, illustrating the main result, is considered afterwards. We then discuss some practical applications and experiments and simulations, related to results of the paper and the results, mentioned after are not original; we need them to provide an experimental justification of Theorem 1 that is the main result of the current paper.

**MATHEMATICAL MODEL**

Consider a segment $J = [0, \mu^*]$ and a $C^2$ smooth function $f(t, z, \mu): \mathbb{R}^{2n+1} \times J \to \mathbb{R}^n$. Suppose that $f(t, z, \mu) \equiv f(t + T(\mu), z, \mu)$. Here the period $T(\mu)$ is a $C^2$ smooth function of the parameter $\mu$, and $T(\mu) > 0$. We may suppose without loss of generality that $T(\mu)$ does not depend on $\mu$, making, if necessary, the transformation $t' = tT(0)/T(\mu)$. Denote

$$z = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}, \quad z_k = \begin{pmatrix} x_k \\ y_k \end{pmatrix},$$

$$k = 1, \ldots, n; \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}; \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix};$$

$$f = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}; \quad \bar{x} = \begin{pmatrix} x_2 \\ \vdots \\ x_n \end{pmatrix}; \quad \bar{y} = \begin{pmatrix} y_2 \\ \vdots \\ y_n \end{pmatrix}; \quad \bar{z} = \begin{pmatrix} z_2 \\ \vdots \\ z_n \end{pmatrix}.$$
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