Chapter 20
Phasing of Traffic Lights in Urban Intersections

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ABSTRACT
In this chapter, the phasing of traffic lights in urban intersections is introduced and discussed applying the graph theory concepts. At first section, some concepts such as traffic streams, vehicle or pedestrian flows, and compatible streams or streams which are in conflict, are introduced. Accordingly, from the viewpoint of graph theory, two approaches are introduced for phasing of traffic streams. The first approach uses the concept of conflict graphs, and the second one uses the concepts such as circular chromatic number and star chromatic number of graphs, developing two procedures of phasing of traffic streams in urban intersections. In addition, these two procedures are compared based on the performance index of total length of complete traffic period which should be optimized.

INTRODUCTION
Consider an urban intersection; a road junction where two or more roads either meet or cross at the same level. Vehicles approaching this intersection prepare themselves to choose a path to traverse the intersection. Vehicles that choose the same path and form the same queue on an approach represent a “traffic stream”. A traffic stream can represent either vehicle or pedestrian flow. An intersection with six traffic streams is presented in Figure 1.

The path used by a traffic stream to traverse the intersection, may cross the one used by another traffic stream. In this case, these streams are in conflict and cannot simultaneously get the right of way. In contrast, when paths used by two different traffic streams do not cross, they are compatible and can get the right-of-way simultaneously. The intersection which is shown in Figure 1, traffic streams 3 and 6 are compatible while traffic streams 2 and 5 are in conflict.

Compatible traffic streams may face the green light at the same time, but those traffic streams which are in conflict with them, must be stopped by red light. A time interval during which each traffic stream gets a turn of green light is a com-
The objective of traffic control by using traffic lights would be to assign to each traffic stream an interval of time during which it has the right of way (when faces a green light) such that the length of a complete traffic period is minimized. Through this chapter, we suppose each interval of green light is of unit length and the right of way can be given only once to any traffic stream.

This problem can be formulated as a graph theoretic model with different approaches. The first approach is based on vertex coloring and the second approach is based on circular coloring of a graph.

The scope of this chapter is limited to an isolated intersection where the influence of other signalized intersections to its performance is negligible (Flammini et al., 2006).

**VERTEX COLORING APPROACH**

In this approach, the intersection is represented by a graph called “conflict graph”. In this graph, each vertex represents a traffic stream and vertices are adjacent if there exists confliction between corresponding traffic streams. (when green light intervals do not overlap). Figure 2(b) presents the conflict graph of the intersection in Figure 2(a).

It is obvious that if we assign 5 different units length green light intervals to 5 traffic streams in Figure 2(a), total length of complete traffic period would be 5 units. But it can be improved by partitioning the graph into minimum number of independent sets with unit length interval of green light. Each independent set consists of compatible traffic streams which can get simultaneous green light interval. The minimum number of independent sets needed to cover all vertices is chromatic number of the graph. Accordingly in Figure 2(b) the chromatic number of conflict graph is 3. A 3-coloring of the graph is presented in Figure 3.

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**Figure 1. An example intersection with 6 traffic streams**

**Figure 2. (a) An intersection with 5 traffic streams (C5) and (b) corresponding conflict graph**