Chapter 7
Theoretical Analysis on Powers-of-Two Applied to JSP: A Case Study of Turbine Manufacturing

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ABSTRACT
This paper discusses the scheduling of precedence-related jobs non-preemptively in a job shop environment with an objective of minimizing the makespan. Due to the NP-hard nature of the scheduling problems, it is usually difficult to find an exact optimal schedule and hence one should rely on finding a near to optimal solution. This paper proposes a computationally effective powers-of-two heuristic for solving job shop scheduling problem. The authors prove that the makespan obtained through powers-of-two release dates lies within 6% of the optimal value. The authors also prove the efficacy of powers-of-two approach through mathematical induction.

INTRODUCTION
Production systems often involve various uncertainties such as unpredictable customer orders or inaccurate estimate of processing times. Managing such uncertainties is becoming critical in the era of “time-based competition” (Luh et al., 1999). The problem of scheduling job shops is widely encountered in many manufacturing industries and still finding methods for improvement. Generating a good job shop schedule for a multi-product manufacturing industry in a reasonable time remains a challenging problem, because of the non-polynomial (NP) hard nature of job scheduling and its inherent computational complexity. Hence it is difficult to apply existing exact algorithms (Depth-First-Search, Branch-and-Bound-First, etc.) to solve the makespan minimization problem of job shop in polynomial time.

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The job shop scheduling problem requires scheduling a set of jobs on a finite set of resources. Each job is a request for the scheduling of a set of operations according to a process routing that specifies a partial ordering among these operations. In order to be successfully performed, each operation requires one or several machines, for each of which there may be several alternatives (e.g., several machines of the same type). Operations are considered to be atomic i.e., once started they cannot be interrupted. In the simplest situation, each operation has a fixed duration, and each resource can only process one operation at a time (Sadeh, 1992).

Generating the best job shop schedule with the makespan criterion for a multi-product manufacturing industry in a reasonable time remains a challenging task, due to its NP hard nature. In this context, Sandeep (2008) developed a priority-based heuristic for minimizing the makespan in a typical turbine manufacturing industry. Krishna (2010) discusses the implementation of an online scheduling support system for a high mix manufacturing firm. Eswaramurthy (2007) presents an application of the global optimization technique called tabu search to the job shop scheduling problem for the problem of finding a minimum makespan.

Though there are many newer approaches to solve job-shop scheduling problems. Some of the recent literature on this is presented in the literature survey section of this paper. However, it is worth noting that most of the problems addressed in the literature are quite generic, they stop at job level assignment to machines. The problem addressed in this paper is a specific type of scheduling problem faced by a turbine manufacturing.

Our initial work on implementing standard meta-heuristics to solve turbine manufacturing problem did not help in getting satisfactory results for the user industry due to the specific nature of business constraints they face. In this regard, we had to develop a new heuristic - by blending the domain/business knowledge with the theoretical knowledge to solve a real world problem.

### JOB SHOP SCHEDULE

An instance of a job shop scheduling problem consists of a collection of jobs \( J = \{j_1, j_2, j_3, \ldots, j_n\} \) which are weighted according to their importance. The time when a job arrives is its release time and is denoted by \( r_j \).

Each job is composed of a set of operations \( O_i = \{o_i^1, o_i^2, o_i^3, \ldots, o_i^k\} \). The processing time associated with each operation is \( P_i = \{p_i^1, p_i^2, p_i^3, \ldots, p_i^k\} \). The execution of each operation \( o_i^k \) requires the use of a set of machines \( M = \{m_1, m_2, m_3, \ldots, m_m\} \). Precedence’s are imposed on the set \( O_i \) using a binary relation \( A \). If \( (u, v) \in A \), then \( u \) has to be performed before \( v \).

Schedule is a function that defines the start time for each operation i.e., \( S(u) \).

A schedule is feasible if,

\[
\forall u \in O : \quad S(u) \geq 0 \\
\forall u, v \in O, \ (u, v) \in A : \quad S(u) + p(u) \leq S(v) \\
\forall u, v \in O, \ (u, v) \notin A, \ M(u) = M(v) : \quad S(u) + p(u) \leq S(v) \\
or \quad S(v) + p(v) \leq S(u)
\]

The length of a schedule is expressed as \( \text{len}(S) = \max_{u \in O} (S(u) + p(u)) \). The goal is to find an optimal schedule, i.e., to find a feasible schedule with minimum length of schedule.

Imposed constraints on the capacity prevent the machines from being allocated to more than one operation at a time. Thus, two different operations \( o_i^k \) and \( o_j^l \) cannot overlap unless they use different machines. An instance of job shop scheduling problem...