Chapter 14

Orbit of an Image Under Iterated System II

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ABSTRACT

An orbital picture is a mathematical structure depicting the path of an object under Iterated Function System. Orbital and V-variable orbital pictures initially developed by Barnsley (2006) have utmost importance in computer graphics, image compression, biological modeling and other areas of fractal geometry. These pictures have been generated for linear and contractive transformations using function and superior iterative procedures. In this paper, the authors introduce the role of superior iterative procedure to find the orbital picture under an IFS consisting of non-contractive or non-expansive transformations. A mild comparison of the computed figures indicates the usefulness of study in computational mathematics and fractal image processing. A modified algorithm along with program code is given to compute a 2-variable superior orbital picture.

1. INTRODUCTION

Let \((X, d)\) be a complete metric space. A map \(g: X \rightarrow X\) is a Banach contraction (also called strictly contractive transformation by Barnsley (2006)) if

\[
d(gx, gy) \leq qd(x, y) \quad \text{for all} \quad x, y \in X, \quad \text{where} \quad 0 \leq q < 1.\]

The map \(g\) is non-expansive if \(q = 1\). A translation map, identity map and isometry are simple examples of non-expansive maps. Some of the properties of contractive maps do not carry over to non-expansive maps. A non-expansive map may not have a unique common fixed point. For example, the identity map on a metric space has every point fixed. Even in a compact space, the sequence of iterates of a non-expansive map
sometimes does not converge to a fixed point. If \( T^n \) for some positive integer \( n \), has a fixed point, it does not necessarily imply that \( T \) has a fixed point. For the theory of contractive and non-expansive operators in non-linear analysis, one may refer to Agarwal, Meehan, and Regan (2001) and Goebel and Kirk (1990).

Orbital pictures are ubiquitous in fractal geometry, as they are always expressed in terms of transformations of an IFS. An orbital picture of an IFS is developed by traversing a defined path, which may be referred to an orbital path. Indeed, a recursive application of an iterative procedure results in an orbital path. However, some issues are of vital concern, while computing the orbit of the picture. For example, there may be some cases when the orbit overlaps. In such cases, to determine the correct orbital picture, one may use the concept of tops union which nicely describes the union of two pictures. Indeed, tops union describes the methodology to take the union of two pictures so as to define a new picture. In other words, tops union simply means, a picture on the top will remain on the top. Moreover, certain real objects are better described by V-variability, as no two clouds are ever the same or two leaves of the same plant differ. V-variability gives a wide range of fractals to the existing deterministic and random fractals, within the framework of Iterated Function Systems. Vast families of homeomorphic objects can be generated with little variation, which may be used to model and study a wide range of phenomena across many areas of science and technology. The concept of V-variability may be used to model almost same looking objects but not exactly the same. For a descriptive knowledge of fractals, V-variable fractals, new generation of fractals and their properties, refer to Barnsley (1993, 2006, 2009), Barnsley, Hutchinson, and Stenflo (2005, 2008), Devaney (1986, 1992), Hutchinson (1981), Mandelbrot (1982), Encarnacao, Peitgen, Sakas, and Englert (1992) and Peitgen, Jürgens, and Saupe (2004).

Orbital pictures for linear and non-linear contractive transformations have initially been studied by Barnsley (2006). Indeed, a semi-group of transformations is needed to generate these beautiful pictures. Orbital pictures are the new mathematical structures, which have been constructed by using one-step feedback process namely, the function iterative procedure. Although, one-step process works very well for contractive transformations, sometimes the problem arises when the transformations are non-contractive, in particular non-expansive transformations but still making a semi-group. In this paper, we generate orbital pictures of different variability for non-contractive and non-expansive transformations using two-step feedback process namely, the superior iterative procedure. It has been observed that the superior iterative procedure generally works very well to construct orbital pictures in case of non-contractive transformations, and converge smoothly wherein one-step process does not converge (Rani, 2010a, 2010b; Rani & Agarwal, 2009a, 2009b, 2010a, 2010b; Rani & Chandra, 2009; Rani & Goel, 2009, 2010; Rani & Kumar, 2002, 2005, 2009, 2003, 2004a, 2004b, 2004c, 2005; Rani & Negi, 2008a, 2008b, 2008c; Rani, Negi, & Mahanti, 2008; Rani & Prasad, 2010; Singh, Jain, & Mishra, 2009, 2011; Singh, Mishra, & Sinkala, 2012; Haq, Sulaiman, & Rani, 2010).

2. PRELIMINARIES

We follow the following notations and definitions from Barnsley (2006), Peitgen et al. (2004), and Singh et al. (2011).

Let \( X \) be a linear metric space. An Iterated Function System (IFS) on \( X \) can be represented by \( F = \{X; f_1, f_2, \ldots, f_M\} \) with \( f_m: X \to X \) being transformations for \( m = 1,2, \ldots, M \) and \( m \geq 1 \).