Chapter 4

A New Method for Ranking Intuitionistic Fuzzy Numbers

Cui-Ping Wei
Qufu Normal University, China

Xijin Tang
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China

ABSTRACT

In this paper the ranking method for intuitionistic fuzzy numbers is studied. The authors first define a possibility degree formula to compare two intuitionistic fuzzy numbers. In comparison with Chen and Tan’s score function, the possibility degree formula provides additional information for the comparison of two intuitionistic fuzzy numbers. Based on the possibility degree formula, the authors give a possibility degree method to rank $n$ intuitionistic fuzzy numbers, which is used to rank the alternatives in multi-criteria decision making problems.

1. INTRODUCTION

Since Zadeh (1965) introduced fuzzy sets theory, some generalized forms have been proposed to deal with imprecision and uncertainty. Atanassov (1986) introduced the concept of an intuitionistic fuzzy set (IFS) characterized by a membership function and a non-membership function. Gau and Buehrer (1993) introduced the concept of vague sets. Bustince and Burillo (1996) showed that vague sets are IFSs. IFSs have been found to be more useful to deal with vagueness and uncertainty problems than fuzzy sets, and have been applied to many different fields.

For the fuzzy multiple criteria decision making (MCDM) problems, the degree of satisfiability and non-satisfiability of each alternative with respect to a set of criteria is often represented by an intuitionistic fuzzy number (IFN), which is an element of an IFS (Liu, 2003; Xu, 2007). The
A New Method for Ranking Intuitionistic Fuzzy Numbers

Comparison between alternatives is equivalent to the comparison of IFNs. Chen and Tan (1994) provided a score function to compare IFNs. Hong and Choi (2000) pointed out the defects and proposed an improved technique based on the score function and accuracy function. Later, Li (2001) and Liu (2003) gave a series of improved score functions. The above functions are called evaluation functions. By using these evaluation functions, we can obtain certain rank of the IFNs. Since IFNs are of fuzziness, the comparison between them may also be expected to reflect the uncertainty of ranking objectively.

In this paper, by extending the possibility degree formula of interval values (Wang, Yang, & Xu, 2005; Xu & Da, 2003) to IFNs, we propose a possibility degree method for ranking IFNs. And the ranking result by the proposed method may reflect the uncertainty of IFSs, and then provide more information to decision makers.

2. POSSIBILITY DEGREE METHOD FOR RANKING INTUITIONISTIC FUZZY NUMBERS

2.1. Possibility Degree Formula for Ranking Two Intuitionistic Fuzzy Numbers and Its Properties

Let \( I = [0,1] \), \( \vee = \max \), \( \wedge = \min \).

Definition 2.1: (Atanassov, 1986) Let \( X \) be an ordinary finite non-empty set. An intuitionistic fuzzy set on \( X \) is an expression given by \( A = \{ (x, u_A(x), v_A(x)) \mid x \in X \} \), where \( u_A : X \to I, \ v_A : X \to I \), with the condition \( u_A(x) + v_A(x) \leq 1 \) for all \( x \in X \); \( u_A(x) \) and \( v_A(x) \) denote, respectively, the membership degree and the non-membership degree of the element \( x \) in \( A \). We abbreviate “intuitionistic fuzzy set” to IFS and represent IFS(\( X \)) the set of all the IFS on \( X \). We call \( \pi_A(x) = 1 - u_A(x) - v_A(x) \) the degree of hesitation (or uncertainty) associated with the membership of element \( x \) in \( A \).

According to Liu (2003) and Xu (2007), for an IFS \( A = \{ (u_A(x), v_A(x)) \mid x \in X \} \), the pairs \( (u_A(x), v_A(x)) \) is called an intuitionistic fuzzy number (IFN). For convenience we denote an IFN by \( (a, b) \), where \( a \in I, \ b \in I, \ a + b \leq 1 \). Let \( Q \) be the set of all the IFNs.

Definition 2.2: (Xu, 2007) Let \( \alpha_i = (a_i, b_i) \in Q, \ i = 1, 2 \), then
1. \( (a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2, b_1 = b_2 \);
2. \( (a_1, b_1) \geq (a_2, b_2) \Leftrightarrow a_1 \geq a_2, b_1 \leq b_2 \);
3. \( (a_1, b_1) \geq (a_2, b_2) \Leftrightarrow a_1 \geq a_2, b_1 \leq b_2 \land (a_1, b_1) = (a_2, b_2) \);
4. \( \alpha_1 = (b_1, a_1) \);
5. \( \alpha_1 + \alpha_2 = (a_1 + a_2 - a_i a_2, b_1 b_2) \);
6. \( \alpha_1 \alpha_2 = (a_1 a_2 + b_1 b_2) \);
7. \( \lambda \alpha_1 = \left( 1 - (1 - a_1)^\lambda, b_1^\lambda \right), \lambda > 0 \);
8. \( a_1^\lambda = \left( a_1^\lambda, 1 - (1 - b_1)^\lambda \right), \lambda > 0 \).

For the practical MCDM problems, experts need to obtain the rank of the alternatives. Suppose the comprehensive evaluation value of each alternative is represented by an IFN \( \alpha \), where \( \alpha = (a, b) \), which indicates the degree of satisfiability and non-satisfiability of each alternative with respect to all the attributes. The larger the degree of hesitation \( \pi(\alpha) \), which is equal to \( 1 - a - b \), the bigger the possible change of the degree of satisfiability and non-satisfiability of the alternative for the experts. As the comprehensive evaluation value is denoted by \( (a, b) \), the degree of satisfiability of the alternative for the experts is actually an interval value written as \( [a, a + \pi(\alpha)] \). Similarly, the degree of non-sat-