Chapter 9
Construction of 3D Triangles on Dupin Cyclides

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ABSTRACT

This paper considers the conversion of the parametric Bézier surfaces, classically used in CAD-CAM, into patched of a class of non-spherical degree 4 algebraic surfaces called Dupin cyclides, and the definition of 3D triangle with circular edges on Dupin cyclides. Dupin cyclides was discovered by the French mathematician Pierre-Charles Dupin at the beginning of the 19th century. A Dupin cyclide has one parametric equation, two implicit equations, and a set of circular lines of curvature. The authors use the properties of these surfaces to prove that three families of circles (meridian arcs, parallel arcs, and Villarceau circles) can be computed on every Dupin cyclide. A geometric algorithm to compute these circles so that they define the edges of a 3D triangle on the Dupin cyclide is presented. Examples of conversions and 3D triangles are also presented to illustrate the proposed algorithms.

1. INTRODUCTION

In CAD, a complex object can be represented by a triangular mesh. This representation gives a lot of freedom for the design, but it needs a large number of topological conditions. The storage and the visualization of complex meshes may be very costly in memory and time, fundamental geometric operations such as intersections may also be very heavy when they are carried on a several complex meshes in the same time.

It would be interesting to have an object formed by 3D triangles having an algebraic, as well as parametric, representation. On each triangle no topological information is needed. The parametric and implicit equations of the 3D
triangles will be used for shape modelling, and to generate some planar approximations for the visualization purposes.

In this work, we propose to compute 3D triangles on a family of algebraic surfaces, called Dupin cyclides. Dupin cyclides are degree 4 algebraic surfaces discovered by the French mathematician Pierre-Charles Dupin in 1822 (Dupin, 1822). These surfaces have a number of properties (e.g., circular lines of curvature, principal circles) that facilitate their use in geometric modelling. The property that lists the families of circles that can be drawn on a Dupin cyclide is of the most importance to the work presented in this paper. Dupin cyclides have three families of circles: the parallels, the meridians and the Villarceau circles. The aim of this work is to build 3D triangles belonging to a Dupin cyclide, where the edges are arcs of parallels, meridians and Villarceau circles.

The paper is organized as follows: in section 2, we recall some definitions and properties of rational quadratic Bézier curves, definitions of rational biquadratic Bézier surfaces and definition and properties of Dupin cyclides. Section 4 presents the proposed algorithm for the construction of 3D triangles on a Dupin cyclide. Section 5 concludes the paper.

2. BACKGROUND

2.1. Rational Bézier Curves and Surfaces

Rational quadratic Bézier curves are second degree parametric curves defined by:

\[ B_0(t) = (1-t)^2 \]
\[ B_1(t) = 2t(1-t) \]
\[ B_2(t) = t^2 \]

and \( w_i, \ i \in \{0, 1, 2\} \), are weights associated with the control points \( P_i \). For a standard rational quadratic Bézier curve, \( w_0 \) and \( w_1 \) are equal to 1, while \( w_i \) can be used to control the type of the conic defined by the curve (Farin, 1993, 1999; Garnier, 2007). As we will model circular arcs using rational quadratic Bézier curve in our algorithm, let us first recall a theorem to compute the weight \( w_i \):

**Theorem 1:** Circle defined by two points and the tangents at these points.

Let \( P_o, P_1, \) and \( P_2 \) be three non-collinear points. \((P_o P_1)\) and \((P_2 P_1)\) are the tangents to the circle \( C \) at \( P_o \) and \( P_2 \). The circle \( C \) has center \( O \) and radius \( R \). Let \( I_2 \) be the middle of \((P_o P_2)\). Let \( P \) be the perpendicular bisector plane of \((P_o P_2)\). Let \( G = \text{bar}(P_o w_o, P_1 w_1, P_2 w_2) \), where \( \text{bar} \) is an abbreviation for barycentre.

The rational quadratic Bézier curve \( y \) of weighted control points \((P_o w_o, P_1 w_1)\) and \((P_2 w_2)\) is an arc of circle if and only if:

\[ \left\| \overrightarrow{O_0 \gamma} \frac{1}{2} \right\| = R \]

which is equivalent to the equation \( \alpha w_1^2 + \beta = 0 \), where \( \alpha \) and \( \beta \) are given by:

\[ \alpha = 4(R^2 - O_0 P_1^2), \quad \beta = w_1^2(R^2 - O_0 G^2) \]
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