Chapter 20
Supporting Teachers’ Instrumental Genesis with Dynamic Mathematical Software

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ABSTRACT
This chapter addresses the need to prepare and support teachers of mathematics in order that they will be able to co-construct with their students classroom environments in which the Standards for Mathematical Practice as well as the content standards (CCSSM, 2010) are implemented fruitfully. Specifically, the chapter describes and illustrates design elements of learning environments with potential to positively impact pre- and in-service teachers’ knowledge of mathematics, facility with technology, and beliefs about how mathematics may be learned. The practice of using appropriate tools strategically is highlighted; however, each of the practice standards is integral to a classroom environment which supports mathematical proficiency (National Research Council, 2001). This chapter examines and illustrates the explicit and intentional instructional design features of using provocative tasks, dynamic technology scaffolding, and sustained intellectual press, which together interact in classrooms to promote the mathematical practices and habits of mind explicated in the CCSSM.

INTRODUCTION
Common Core State Standards for Mathematics (2010) lays out a vision for students to be mathematically proficient as discussed in Adding It Up, with the strands of mathematical proficiency including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition (National Research Council, 2001). Students are encouraged to develop productive mathematical habits of mind (Cuoco, Goldenberg, & Mark, 1996). Students
in schools are expected to practice and exhibit standards of mathematical practice and knowledge of mathematical content. To achieve these kinds of goals, students benefit from feeling invited into the powerful world of mathematical thinking and reasoning when they come to recognize that these goals are not deterministic and achievable as entities. Rather, successful mathematical learners will be those who develop dispositions to see their progress in mathematics as a process of coming to know, rather than knowing or somehow being done. As learners come to know mathematics in ways that are sensible to them, myriad ways of thinking, representing, computing, exploring, connecting, and communicating are beneficial. The use of digital tools to support learners coming to know affords possibilities that otherwise may not exist.

Through repeated studies with pre-service and in-service teachers, my research has suggested that under the conditions that will be discussed in this paper, teachers may come to see the use of digital dynamic cognitive tools (DCT) (e.g., Fathom, TinkerPlots, Cabri II Geometry, Geometer’s Sketchpad, CPMP-Tools, GeoGebra, TI-Nspire) as well as other tools as increasingly valuable for supporting the mathematical learning of a wide range of learners. In light of CCSSM, if students are expected to use tools strategically and become mathematically proficient, it is imperative that their teachers are given an opportunity to do so as well.

The reality of living in the 21st century is that digital technology is rapidly evolving and will likely continue to do so. Research literature suggests that many mathematics teachers do not use technology with students or allow students to use technology in their classrooms for a host of reasonably predictable reasons such as personal concerns related to their role as teacher, management concerns related to managing the classroom as well as student learning, and technology concerns related mostly to personal facility with tools (Zbiek & Hollebrands, 2008). I make a distinction about digital technology in this chapter to mean DCT as opposed to digital tools like social media, smartboards, document cameras, and the like. Like Zbiek, Heid, Blume, and Dick (2007), I assume a potentially synergistic relationship between technical and conceptual mathematical activity in a technology-rich environment.

According to Pea (1985), “A cognitive technology is provided by any medium that helps transcend the limitations of the mind, such as memory, in activities of thinking, learning, and problem solving” (p. 168). DCTs may include rudimentary technologies such as pipe cleaners and straws, geostrips and other physical tools used to explore mathematical relationships as well as computer- or calculator-based dynamic tools such as TI-Nspire, Fathom 2, TinkerPlots, Cabri II Geometry, Cabri 3D, Geometers’ Sketchpad, GeoGebra, Core Math Tools, and online applets, to name a few. I see all of these as DCT with potential to support learners’ technical and conceptual mathematical activity (Zbiek, et al., 2007). Furthermore, the non-digital DCT can serve as important scaffolding devices for the digital DCT. This will become more apparent in later sections.

Over the past 15 years of my career, I have had the honor of working closely with pre- and in-service teachers in rural and urban settings as well as undergraduate and graduate students pursuing mathematics education-related careers. Through this work and associated research, I have come to more fully understand some of the complexities of supporting the work of mathematics teachers. In the mix of considerations, and perhaps at the heart of the work presented in this paper, lies the big question of, “What does it mean to learn mathematics?” Related questions include: Who gets to learn mathematics? How do they get to engage with mathematical ideas? How will someone determine whether another person has learned something? I summarize these questions frequently by: Who is learning? What are they learning? and, How do you know? These questions transcend learning environments and