A Survey and Comparison of Optimization Methods for Solving Multi-Stage Stochastic Programs with Recourse

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ABSTRACT

In the last decade, multi-stage stochastic programs with recourse have been broadly used to model real-world applications. This paper reviews the main optimization methods that are used to solve multi-stage stochastic programs with recourse. In particular, this paper reviews four types of optimization approaches to solve multi-stage stochastic programs with recourse: direct methods, decomposition methods, Lagrangian methods and empirical-distribution methods. All these methods require some form of approximation, since multi-stage stochastic programs involve the evaluation of random functions and their expectations. The authors also provides a classification of the considered optimization methods. While decomposition optimization methods are recommendable for large linear problems, Lagrangian optimization methods are appropriate for highly nonlinear problems. When the problem is both highly nonlinear and very large, an empirical-distribution method may be the best alternative.

Keywords: Decomposition Optimization Methods, Direct Optimization Methods, Empirical-Distribution Optimization Methods, Lagrangian Optimization Methods, Multi-Stage Stochastic Programs with Recourse, Optimization Methods

INTRODUCTION

Decision-making processes usually require the development of an active approach to uncertainty: model it rather than ignore it. The standard approach is the modeling of uncertain quantities by random variables. Probabilistic models allow both the analysis of problems with uncertainty by means of mathematical methods and the development of robust solution techniques.

In the last decade, many real-world applications: optimal investment strategies (Birge, 1995), real option valuation (Kallio, 2002), capacity expansion planning, power system economics (Wang and Kong, 2010), batch plant...
scheduling (Wang et al., 2009), economic policy planning and telecommunication network planning (Higle & Sen, 1996), among many others, have been modeled by the use of multi-stage stochastic programs with recourse due to the powerful characteristics of this type of modeling. Recently, Thiele et al. (2009), Chen and Banet (2010), Dyer and Stougie (2006) and Swamy and Shmoys (2005) present good discussions about complexity issues of the multi-stage stochastic programs with recourse.

One of the main characteristics of the multi-stage stochastic program with recourse is that the first-period decision is independent of which second-period scenario actually occurs. This characteristic, called the non-anticipativity property, allows splitting the multi-stage stochastic program with recourse in a sequence of easier decision problems, making possible the use of efficient optimization methods.

This paper reviews four types of optimization approaches to solve multi-stage stochastic programs with recourse: direct methods, decomposition methods, Lagrangian methods and empirical-distribution methods. All methods require some form of approximation, since multi-stage stochastic programs involve the evaluation of random functions and their expectations. We recognize that the empirical approach may essentially apply to all the other methods, without being really a distinct approach. However, we incorporate it separately to emphasize some particular advantages of directly estimating probability distributions.

The rest of the paper is organized as follows. The second section describes the mathematical formulation of multi-stage stochastic programs with recourse. The following section reviews four types of optimization approaches to solve multi-stage stochastic programs with recourse: direct methods, decomposition methods, Lagrangian methods and empirical-distribution methods. Finally, the last section presents the paper conclusions and provides a bi-dimensional (based on two criteria) classification of the considered optimization methods.

**MULTI-STAGE STOCHASTIC PROGRAM WITH RECOURSE**

To understand the multi-stage stochastic program with recourse, we begin by presenting an easier version of this model: the two-stage stochastic linear program with recourse.

**Two-stage Stochastic Linear Program with Recourse**

Consider the following deterministic linear programming:

\[
\begin{align*}
\text{Min}_x & \quad c x \\
\text{s.t.} & \quad A x = b \\
& \quad T x = \omega \\
& \quad x \geq 0
\end{align*}
\]

Such models are applied to analyze a wide variety of practical problems although they assume that all parameters are known (deterministic). For instance, the constraint \(T x = \omega\) can represent the future demand meeting requirement for certain goods. However, in that case, the exact amount of this demand is unknown at the time that decisions need to be made.

If we consider the right-hand side vector, \(\omega\), as a random variable, then the previous model is not well defined. A common way to extend the deterministic model to random variables is by assuming that the decision variable, \(x\), is specified before knowing the realization of \(\omega\) and, once the realization is known, deviations \(\omega - T x\) have to be corrected at minimum costs by solving simple linear programs. The costs of these corrections (or recourse actions) are given by a recourse cost function, \(v(\omega - T x)\), and are computed for each possible realization of the random parameter \(\omega\). Thus, the criterion on choosing \(x\) is the minimal total expected cost, consisting of the sum of the direct cost, \(c x\), and the expected recourse cost, \(E_{\omega} [v(\omega - T x)]\). This model, known as the two-stage stochastic linear program with recourse, is commonly formulated as follows, see (Birge, 1997):
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