Chapter 2

Numerical Methods of Multifractal Analysis in Information Communication Systems and Networks

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ABSTRACT

In this chapter, the main principles of the theory of fractals and multifractals are stated. A singularity spectrum is introduced for the random telecommunication traffic, concepts of fractal dimensions and scaling functions, and methods used in their determination by means of Wavelet Transform Modulus Maxima (WTMM) are proposed. Algorithm development methods for estimating multifractal spectrum are presented. A method based on multifractal data analysis at network layer level by means of WTMM is proposed for the detection of traffic anomalies in computer and telecommunication networks. The chapter also introduces WTMM as the informative indicator to exploit the distinction of fractal dimensions on various parts of a given dataset. A novel approach based on the use of multifractal spectrum parameters is proposed for estimating queuing performance for the generalized multifractal traffic on the input of a buffering device. It is shown that the multifractal character of traffic has significant impact on queuing performance characteristics.

INTRODUCTION

Often in telecommunication applications, the measured characteristics of traffic datasets display stochastic self-similar properties (i.e. fractality). Here it is assumed that a measure of similarity is the traffic type with appropriate amplitude normalization. Accurate structural observation is complicated for datasets, self-similarity however allows for considering the stochastic nature of many network devices and events, which jointly influence the network traffic. One value suffices
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for the quantitative description of fractals (i.e. the Hausdorff dimension or a scaling index) describing an invariance of geometry or statistical performances at a given level of rescaling. However in the fields of physics, chemistry, biology, and telecommunications, there are many appearances, which demand propagation of the fractal concept on complicated structures with more than one scaling index. Such structures are often characterized by a whole spectrum of indices and Hausdorff dimension is only one of them. Complex fractals, also known as multifractals, are important because they as a rule occur in nature, whereas simple self-similar objects represent idealization of real appearances. Actually, employment of the multifractal approach means that the studied object somehow can be divided into parts, each having its own self-similar properties.

Thus multifractals are non-homogeneous fractal objects, for which complete description is required, unlike the regular fractals, there is not enough information in any one value of fractal dimension, but a whole spectrum of such dimensions is required, the number of which, generally speaking, is infinite. The distinctive feature of the latter consists in the fact that, along with the global characteristics of stochastic processes (obtained as a result of the procedure of averaging on large time intervals), allow for considering singularities of their local structure. Their versatility is in important techniques based on fractal representations and wavelet transforms.

The material in this chapter is divided into three parts. The first part sets out the basic theory of fractals and multifractals, as well as methods of determining the basic parameters of multifractal processes using wavelet transforms. The other two parts deal with specific technical tasks, where investigation of multifractal properties of the processed sequences yield innovative solutions and algorithms. The second part is devoted to the use of fractal analysis for problems of detection of traffic anomaly, which allows for a fundamentally new approach to algorithms development. In the third part, for the generalized multifractal traffic the new practical evaluation method of telecommunication networks queuing performance is offered.

### THEORY OF FRACTALS AND MULTIFRACTALS

The term “fractal” was used for the first time in Benoît Mandelbrot’s work (Mandelbrot, 1982). The word fractal is derived from the Latin fractus meaning “fractured” or “broken.” Mandelbrot used the term “fractals” for geometric objects that have strongly fragmented shape and can possess the property of self-similarity. It is possible to generalize the concept of fractal to any object (image, speech, telecommunication traffic, etc.) some parameters of which are remain invariant with change in scale or time. Thus, the principal property of such objects (i.e. self-similarity) implies that at augmentation, its parts are similar (in some specified sense) to its total shape.

The property of exact self-similarity is a characteristic of the regular fractals only. If an element of randomness is to be included in the algorithm of their creation instead of the determined method of construction (as it happens, for example, in many processes of diffusion growth of clusters, voltage failure, etc.), then the so-called incidental fractals appear. Their basic difference from regular ones is that the property of self-similarity holds true only after a corresponding averaging on the base of all statistically independent realizations of the object. For quantitative description of fractals, a single value is enough - a fractal dimension (Hausdorff dimension) or the index of scaling which is determined as follows

\[
D_f = -\lim_{\varepsilon \to 0} \frac{\ln M(\varepsilon)}{\ln \left(\frac{1}{\varepsilon}\right)}
\]

(1)