Chapter 5
Nonlinear is Essential,
Linearization is Not
Enough, Visualization is
Absolutely Necessary

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ABSTRACT
Linear differential equations have been well understood for some time and are an important tool for studying the nonlinear systems that most frequently arise in mathematical models of real world systems. Nonlinear systems do not usually have formula solutions, but with graphics, we can see the behaviors of the solutions and thereby “understand” the differential equations. Dynamic and interactive presentations provide students with a streamlined route to understanding these behaviors, resulting in immense power and efficiency that was not previously available at the undergraduate level.

INTRODUCTION
Mathematical models of real world problems are almost always nonlinear. We can linearize many such models to give approximate solutions, but these approximations are only good in a sufficiently small neighborhood of an equilibrium point. However, we show that often what at first glance appears to be an appropriately small neighborhood is not at all sufficient for accurate long-term prediction, which is usually the objective.

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Our examples will be from Differential Equations (DEs), which provide a vital connection between mathematics and the real world. We proceed from the most elementary to more advanced examples, showing how dynamic graphic software helps students to see the behavior, to know when and where to trust a linear approximation, as well as what to do when you cannot. We get the basic information in two-dimensional systems from the $xy$ phase plane representation, but students need to understand the connection to the solution graphs, $tx$ and $ty$. We also sketch advantages and issues of graphic software for higher order differential equations.

What is of utmost importance for DEs is not a formula for the solutions, which usually does not even exist for a nonlinear DE, but rather the behaviors of the solutions, which are easily absorbed through their pictures (created by numerical methods). We must attend, of course, to the issue of when can we trust such pictures.

Furthermore, we need different kinds of dynamic graphic software. First, to convey concepts, we have found immense benefit in interactive illustrations that provide guided exploration of good examples. The package on which we rely is Interactive Differential Equations (Hohn et al, 2000). Second, to realize the power that the graphic revolution has unleashed, we need open-ended solvers that allow students to enter their own DEs for graphic experimentation; for this, we have always used MacMath (Hubbard & West, 1995, 1999).

We shall begin with the background of DEs with a graphic interface, then review the linear behaviors that are well-understood. Next, with examples, we show the complications that arise in nonlinear DEs, and why linearization about the equilibria does not completely explain the behaviors. We extend the discussion to include chaotic behavior and the assistance provided by software that deals with higher dimensions. Lastly, we discuss the two types of software and what remains to be done for the open-ended version to be useful.

**BACKGROUND**

**Differential Equations and their Graphs**

We shall concentrate in this chapter on two-dimensional systems of autonomous differential equations, of the form

$$
\frac{dx}{dt} = f(x, y), \quad \frac{dy}{dt} = g(x, y). \quad (1)
$$
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The e-Learning Puzzle in Turkey: Déjà Vu?
Selçuk Özdemir (2010). *Teaching Cases Collection* (pp. 143-156).
www.igi-global.com/chapter/learning-puzzle-turkey/40573?camid=4v1a