Chapter 8
Applets for Mathematical Learning

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ABSTRACT
The purpose of this chapter is to describe dynamic interactive applets, which are available to all students, to assist in the understanding of sophomore-level engineering mathematics. The topics illustrated here with screen shots are the partial differential equations for heat conduction in one and two space dimensions, the wave equation and Laplace equation in two space dimensions, systems of three ordinary differential equations, and the display of vector fields in three dimensions. An applet is also shown for iterated function systems, although this is not usually part of the engineering sequence. Additional applets mentioned here and available online include the wave equation in one space dimension and two-by-two matrices. The purpose of these applets, all written by the author and freely available, is to help students experiment visually and dynamically with the mathematical concepts.

INTRODUCTION
It is common knowledge that abstraction is hard to understand if the learner is not equipped with the proper background. Many years of teaching engineering students have led me to conclude that mathematics students have this problem with partial differential equations and Fourier series, because in many cases they have little mechanical experience. For years when teaching the heat equation, for example, I described the heating and cooling of iron bars at the blacksmith’s forge, and the changing of the colors with temperature, and this did not help the students very
much. I could talk about something closer to their experience, such as the cooling of pizza. This seemed to be more effective. With the wave equation it is possible to discuss the vibrations of a musical instrument in order to introduce Fourier series. This topic is very real to the string musicians in the class who have experienced the higher harmonics with their own eyes and ears. Even so, this discussion does not usually convey to all students the mental pictures which are needed for understanding the mathematics. For this reason, writing some dynamic software to help students experiment with the ideas of these equations in a visual manner seemed imminent. These are interactive java applets, and the main purpose is to let students explore the mathematics visually as a supplement to whatever textbook is being used. Very little background is required of the student. This chapter describes these applets. We show pictures from some of them, and discuss the mathematical context. Of course, one cannot see the full power of the dynamical environment unless the applets are actually utilized.

**The Heat Equation in One Space Dimension**

The heat equation describes the flow of energy due to conduction from hot regions to colder regions of static material. For material which can be considered one-dimensional, the temperature $u$ is a function of time $t$ and position $x$ along an interval.

The typical undergraduate course shows how to solve the equation in one space dimension using separation of variables. Although this computation introduces many important ideas, it does not help all students to get a sufficient understanding of the meaning of the equation. So I have developed an applet which allows students to explore the possibilities.

In Figure 1, several screen shots are shown for a case in which the initial temperature is piecewise constant. The smoothing property of the heat equation is obvious when you watch it run. Specifically, the applet has a “Stop” button, but no matter how fast the student tries to click the button the solution already looks smooth. The boundary values in Figure 1 are 0 degrees at one end and -4 at the other. The temperature is displayed both as a graph and using colors to indicate temperature. Colors in the applet range from blue at the coldest parts to black to orange at the hottest. Because this is dynamic software, the student can see the colors and graph change with time. The comparison of the graph with the colors is also an important feature for students because most introductory mathematics texts emphasize the graphs of functions, while in many applications they ought to be thinking of the domain of the function and what is happening there. The colors are drawn in the material, the domain in which the heat conduction occurs. The student can explore both initial and boundary conditions in this applet. For example initial conditions can be typed
Teaching What We Don’t Know: Failing to Adequately Prepare Teachers to Use Technology for the Benefit of Students with Special Needs
Joy E. Harris (2012). *Communication Technology for Students in Special Education and Gifted Programs* (pp. 320-327).
www.igi-global.com/chapter/teaching-don-know/55484?camid=4v1a

Teaching Social Studies With Games
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