Chapter VII
3D Reconstructions from Few Projections in Oral Radiology

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ABSTRACT

Established techniques for three-dimensional radiographic reconstruction such as computed tomography (CT) or, more recently cone beam computed tomography (CBCT) require an extensive set of measurements/projections from all around an object under study. The x-ray dose for the patient is rather high. Cutting down the number of projections drastically yields a mathematically challenging reconstruction problem. Few-view 3D reconstruction techniques commonly known as “tomosynthetic reconstructions” have gained increasing interest with recent advances in detector and information technology.

INTRODUCTION

Three-dimensional (3D) imagery has been gaining ever increasing attention during the last decade. Since human beings are familiar with a 3D world surrounding them, representing image information also in 3D is a natural desire. This holds true particularly in medical radiography, where more or less complex structures are visualized for the purpose of diagnosis or sophisticated treatment procedures. Established techniques for radiographic 3D reconstruction such as Computed Tomography (CT) including its most recent extension, Cone Beam CT (CBCT) are based on an extensive set of projections from all around the object. Also, the imaging geometry of each and every projection has to be precisely known a priori, requiring large-scale scanners and sophisticated hardware technology. Obviously, since the x-ray dose is directly related to the number of
projections, techniques using multiple projections administers a rather high dose to the patient. In Germany the increasing patient dose over the last decade is attributed to the increasing number of CTs (Bundesamt für Strahlenschutz, 2004). Dose considerations as well as other practical aspects such as flexibility, costs or availability have been the driving forces for the development of alternative techniques. If one drastically cuts down the number of input projections, then obviously the dose will also be reduced considerably. Such an approach, however, poses major challenges for the 3D reconstruction process, since the lack of input information renders the reconstruction problem at least instable. The reconstruction problem itself is a classical “inverse problem”, where the results of actual observations (i.e. the projection data) are used to infer the values or the parameters characterizing the system under investigation. This chapter will summarize the most important techniques to tackle the very challenging problem of few-view 3D reconstruction. To begin with, we shall briefly resume the physical principles of radiographic image formation and the mathematical background of established 3D imaging techniques, such as CT.

**RADIOGRAPHIC PROJECTION AND 3D RECONSTRUCTION FROM MULTIPLE PROJECTIONS**

The projection value measured by any x-ray sensitive receptor follows the well-known Lambert-Beer law:

\[
I_1 = I_0 e^{-\mu(x, y, z)dl}
\]  

(1)

with \(I_1\) defining the intensity behind an absorber and \(I_0\) the input intensity, respectively. The parameters \(\mu\) and \(d\) denote the mass absorption coefficient and the thickness of the absorber. We aim to estimate \(\mu\) as shade of gray at discrete instances to obtain a reliable representation of the object.

In 1917, the Austrian mathematician Johann Radon discovered that the two-dimensional (2D) distribution of properties of an object may be obtained from an infinite number of line integrals sampled through the object. Mathematically, a function \(f(x, y)\) can be completely described by the complete number of straight line integrals through the support of \(f(x, y)\), i.e.

\[
f(x, y) = \int_{-\infty}^{+\infty} f(x(l), y(l)) dl
\]  

(2)

The famous method predominantly applied for image reconstructions in CT-scanners uses this formula in a process termed “Filtered Back-projection (FBP)”.

\[
P(\Theta, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \delta(x \cos \Theta + y \sin \Theta - t) dx dy
\]  

(3)

where \(\Theta\) denotes the projection angle and \(t\) the detector position in the beam (Fig. 1). Diracs delta function \(\delta\) is required to define the line interval. The CT image reconstruction problem is to compute \(f(x, y)\) given \(P(\Theta, t)\). Note, that the term “\(x \cos \Theta + y \sin \Theta - t\)” in equation (3) represents a line equation (e.g. green “ray-line” in Fig. 1), (Beyerer & Leon, 2002) the sum of which the integrals are sampled. In other words, the measured data on the image receptor represent the integrals over a finite number of lines connecting the x-ray source with the detector cells.

We can easily see from eqs. (2) and (3), that the inversion of the Radon transform, i.e. the inverse Radon transform, which is a standard procedure in CT, in theory requires an infinite number of line integrals (photon counts) to be measured. A good approximation, however may also be obtained...
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