ABSTRACT

The aim of this paper is to introduce the notion of anti fuzzy (prime) deductive system in BL-algebra and to investigate their properties. It is shown that the set of all deductive systems (with the empty set) of a BL-algebra $X$ is equipotent to a quotient of the set of all anti fuzzy deductive systems of $X$. The anti fuzzy prime deductive system theorem of BL-algebras is also proved.

Keywords: Algebraic Structure, Anti-Fuzzy Deductive System, BL-Algebra, Empty Set, Prime

1. INTRODUCTION

In Hajek (1998), Hajek introduced the basic logic algebra (BL-algebra) as the algebraic structure of his basic logic. Up to now, that algebra have been widely studied and emphasis have been put on filter theory (Kondo & Dudek, 2008; Turunen, Tchikapa, & Lele, 2012; Turunen, 2001). It is well known that in various logical systems,

the theory of ideals and filters plays a fundamental role, ideals or filters correspond to sets of provable formulas and closed with respect to modus ponens. This is to say that ideals and filters are not just abstract concepts, but are mathematically deep and significant concepts with applications in various areas. In BL-algebras, Turunen (2001) called them deductive systems since they are not lattice filters.

Fuzzy deductive systems are useful tools to obtain results on classical deductive systems in BL-algebras. The fuzzification of deductive systems in BL-algebras has been investigated by many researchers (Jeong, 1999; Kondo & Dudek, 2005; Liu & Li, 2005; Tchikapa & Lele, n.d.). In this paper, we introduce the notions of anti fuzzy deductive system and anti fuzzy prime deductive system in BL-algebras. Using the notion of lower t-level set of a fuzzy set, we establish the transfer principle for these two notions. We also prove that $F(X) \cup \{\Phi\}$ is equipotent to $AFF(X)/R_t$, where $F(X)$, $AFF(X)$ and $R_t$ are the set of all the deductive systems of a BL-algebra $X$, the set of all anti fuzzy deductive systems of $X$ and an equivalence relation on $AFF(X)$ respectively. Moreover we prove the anti fuzzy prime deductive system theorem.

DOI: 10.4018/jalr.2012070103

Copyright © 2012, IGI Global. Copying or distributing in print or electronic forms without written permission of IGI Global is prohibited.
2. PRELIMINARIES

We recollect some definitions and results which will be used in the following and we shall not cite them every time they are used.

Definition 2.1: A BL-algebra is an algebra \((X, \wedge, \lor, *, \to, 0, 1)\) of type \((2; 2; 2; 2; 0; 0)\) that satisfies the following conditions:

BL-1: \((X, \wedge, \lor)\) is a bounded lattice;

BL-2: \((X, *, \to)\) is a commutative monoid, i.e., * is commutative and associative with \(x * x = 1\);

BL-3: \(x * y \leq z\) iff \(x \to y \to z\) (Residuation);

BL-4: \(x * y \leq z\) iff \(x * y \leq z\) (Divisibility);

BL-5: \((x \to y) \vee (y \to x) = 1\) (Prelinearity).

Example 2.1:
1. Let \(X\) be a nonempty set and let \(P(X)\) be the family of all subsets of \(X\). Define operations \(*\) and \(\to\) by:
   \[ A * B = A \cap B \quad \text{and} \quad A \to B = A^c \cup B \quad \text{for all} \quad A, B \in P(X) \]
   respectively. Then \(P(X), \wedge, \lor, *, \to, 0, 1\) is a BL-algebra called the power BL-algebra of \(X\).
2. \(L = [0;1], \wedge, \lor, *, \to, 0, 1\) where \(\to\) is the residuum of a continuous t-norm * is a BL-algebra.

The following properties hold in any BL-algebra:

Lemma 2.1:
1. \(x \leq y\) iff \(x \to y = 1\);
2. \(x \to (y \to z) = (x * y) \to z\);
3. \(x * y \leq x \land y\);
4. \((x \to y) * (y \to z) \leq x \to z\);
5. \(x \lor y = ((x \to y) \land (y \to x)) \to x\);
6. \(x \to (y \to z) = y \to (x \to z)\);
7. \((x \lor y) \to z = (x \to z) \land (y \to z)\);
8. \(x \to y \leq (y \to z) \to (x \to z)\);
9. \(y \to x \leq (z \to y) \to (z \to x)\);
10. if \(x \leq y\) then \(y \to z \leq x \to z\) and \(z \to x \leq z \to y\);
11. if \(x \lor x^- = 1\) then \(x \land x^- = 0\) where \(x^- = x \to 0\);
12. \(y \leq (y \to x) \to x\);
13. \(x \leq y \to (x * y)\);
14. \(x * (x \to y) \leq y\);
15. \(1 \to x = x; \quad x \to x = 1; \quad x \to 1 = 1; \quad x \leq y \to x\);

We briefly review some fuzzy logic concepts, we refer the reader to Liu and Li (2005), and Zadeh (1965) for more details.

Definition 2.2: Let \(X\) be a BL-algebra. A fuzzy subset of \(X\) is a function \(\mu : X \to [0;1]\)

Definition 2.3:
1. A deductive system (ds for short) of a BL-algebra \(X\) is a subset \(F\) containing 1 such that \(x \to y \in F\) and \(x \in F\) imply \(y \in F\).
2. A prime ds is a proper ds such that \(x \land y \in F\) implies \(x \in F\) or \(y \in F\), \(\forall x, y \in X\).

Definition 2.4: A fuzzy subset \(\mu\) of \(X\) is a called:
1. A fuzzy ds if \(\mu(1) \geq \mu(x)\) and \(\mu(y) \geq \min\{\mu(x \to y) ; \mu(x)\}\), \(\forall x, y \in X\).
2. A fuzzy prime ds if it is non constant and \(\mu(x \lor y) \geq \min\{\mu(x) ; \mu(y)\}\), \(\forall x, y \in X\).

The complement of a fuzzy set \(\mu\) on \(X\) will be denoted by \(\overline{\mu}\) and is defined by \(\overline{\mu}(x) = 1 - \mu(x)\).

The following theorem gives a characterization of fuzzy ds.
Related Content

Lost Student Tracking in an Incomplete and Imprecise Information Environment Using Soft Computing Paradigm
Bio-Inspired Algorithms for Text Summarization: A Review
www.igi-global.com/chapter/bio-inspired-algorithms-for-text-summarization/177982?camid=4v1a

Diversity Conserved Chaotic Artificial Bee Colony Algorithm based Brightness Preserved Histogram Equalization and Contrast Stretching Method

Correctness of Self-Stabilizing Algorithms Under the Dolev Model When Adapted to Composite Atomicity Models
Chih-Yuan Chen, Cheng-Pin Wang, Tetz C. Huang and Ji-Cherng Lin (2012). International Journal of Artificial Life Research (pp. 16-31).
www.igi-global.com/article/correctness-of-self-stabilizing-algorithms-under-the-dolev-model-when-adapted-to-composite-atomicity-models/101292?camid=4v1a