Chapter 24

Non Linear and Non Gaussian States and Parameters Estimation using Bayesian Methods—Comparatives Studies

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ABSTRACT

This chapter deals with the problem of non-linear and non-Gaussian states and parameters estimation using Bayesian methods. The performances of various conventional and state-of-the-art state estimation techniques are compared when they are utilized to achieve this objective. These techniques include the Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), and Particle Filter (PF). In the current work, the authors consider two systems (biological model and power system) to perform evaluation of estimation algorithms. The results of the comparative studies show that the UKF provides a higher accuracy than the EKF due to the limited ability of EKF to accurately estimate the mean and covariance matrix of the estimated states through linearization of the nonlinear process model. The results also show that the PF provides a significant improvement over the UKF because, unlike UKF, PF is not restricted by linear-Gaussian assumptions which greatly extends the range of problems that can be tackled.

INTRODUCTION

Many process operations, such as modeling, monitoring, and control, require the availability of state and/or parameter measurements. However, due to the difficulty of, or cost associated with, obtaining these measurements, state and/or parameter estimators are often used to overcome this problem. For example, in process monitoring and control, sometimes it is challenging to measure some of the key variables. In such cases, estimates of these variables can

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be obtained using state estimation. Also, in modeling, several model parameters need to be estimated. Estimating these parameters usually requires several experimental setups that can be challenging and expensive. Hence, estimating such parameters using state estimation can be of great value. In this chapter, the objective is to compare the performances of various state-of-the-art state estimation techniques in estimating the state variables using different kinds of observation models (i.e., biological model and power system) and their abilities to estimate some of the key system parameters, which are needed to define the process model. Several estimation techniques, such as the extended Kalman filter, unscented Kalman filter and more recently the Sequential Monte Carlo method have been developed and utilized in many applications. Several estimation techniques, such as the extended Kalman filter, unscented Kalman filter and more recently the Sequential Monte Carlo method have been developed and utilized in many applications. The classical Kalman Filter (KF) was developed in the 1960s (Kalman, 1960), and has been widely applied in various engineering and science areas, including communications, control, machine learning, neuroscience, and many others. In the case where the model describing the system is assumed to be linear and Gaussian, the KF provides an optimal solution (Simon, 2006; Grewal & Andrews, 2008). KF has also been formulated in the context of Takagi-Sugeno fuzzy systems, which can be described by a convex set of multiple linear models (Chen et al., 1998; Simon, 2003). It is known that KF is computationally efficient; however, it is limited by the non-universal linear and Gaussian modeling assumptions. To relax such assumptions, the Extended Kalman Filter (Simon, 2006; Grewal & Andrews, 2008; Julier et al., 1997; Ljung et al., 1979; Kim et al., 1994) and the Unscented Kalman Filter (Simon, 2006; Grewal & Andrews, 2008; Wan et al., 2000; Wan et al., 2001; Sarkka et al., 2001; Sarkka et al., 2007) have been developed. In extended Kalman filtering, the model describing the system is linearized at every time sample (which means that the model is assumed to be differentiable). Therefore, for highly nonlinear models, EKF does not usually provide a satisfactory performance. The UKF, on the other hand, instead of linearizing the model to approximate the mean and covariance matrix of the state vector, uses the unscented transformation to approximate these moments. In the unscented transformation, a set of samples (called sigma points) are selected and propagated through the nonlinear model to improve the approximation of these moments and thus the accuracy of state estimation. Other state estimation techniques use a Bayesian framework to estimate the state and/or parameter vector (Beal et al., 2003). The Bayesian framework relies on computing the probability distribution of the unobserved state given a sequence of the observed data in addition to the state evolution model. Consider an observed data set, which is generated from a model defined by a set of unknown parameters, \( z \) (Smidl et al., 2005). The beliefs about the data are completely expressed via the parametric probabilistic observation model, \( P(y|z) \). The learning of uncertainty or randomness of a process is solved by constructing a distribution \( P(z|y) \), called the posterior distribution, which quantifies our belief about the system after obtaining the measurements. According to Bayes rule, the posterior can be expressed as

\[
P(z|y) = \frac{P(y|z)P(z)}{P(y)}
\]  

where \( P(y|z) \) is the conditional distribution of the data given the model parameter vector, \( z \), which is called the likelihood function, and \( P(z) \) is the prior distribution, which quantifies our