Chapter 16
Soft Sets: Theory and Applications

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ABSTRACT

This chapter is about soft sets. A brief account of the developments that took place in last 14 years in the field of Soft Sets Theory (SST) has been presented. It begins with a brief introduction on soft sets and then it describes many generalizations of it. The notions of generalized fuzzy soft sets are defined and their properties are studied. After that, a notion of mapping, called soft mapping, in soft set setting is introduced. Later, algebraic structures on soft sets like soft group, soft ring, etc. are discussed. Then the next section deals with the concept of topology on soft sets. Here two notions of topology in soft sets are introduced, which are the topology of soft subsets and the soft topology, respectively. The idea of entropy for soft sets is defined in the later section. Next, some applications of hybrid soft sets in solving real life problems like medical diagnosis, decision-making, etc. are shown. Issues like measurement of similarity of soft sets are also addressed.

INTRODUCTION

Uncertainty is a common phenomenon of our daily existence because our world is full of uncertainties. In our daily life we encounter many situations where we do take account of these uncertainties. Therefore it is natural for man to understand and try to model this uncertainty prevailing in physical world. From centuries, in almost all branches of Science or in Philosophy, attempts have been made to understand and represent the features of uncertainty. Perhaps that is the main reason behind the development of Probability theory and Stochastic techniques which started in early eighteenth century. Till mid-twentieth century, Probability theory was the only tool for handling
certain type of uncertainty called “Randomness”. But there are several other kinds of uncertainties; one such type is called “vagueness” or “imprecision” which is inherent in our natural languages. In 1965, L. A. Zadeh [Zadeh, 1965] coined his remarkable theory of Fuzzy sets that deals with a kind of uncertainty known as “Fuzziness” and which is due to partial membership of an element in a set. Later this “Fuzziness” concept leads to the highly acclaimed theory of Fuzzy Logic. This theory has been applied with a good deal of success to many areas of engineering, economics, medical science etc., to name a few, with great efficiency.

After the invention of fuzzy sets many other hybrid concepts begun to develop. In 1983, K. Atanassov [Atanasov, 1986] introduced the idea of Intuitionistic fuzzy sets, a set with each member having a degree of belongingness as well as a degree of non-belongingness. This is again a generalization of fuzzy set theory. Although Fuzzy set theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all sorts of uncertainties prevailing in different real physical problems. Thus search for new theories has been continued. In 1982, Z. Pawlak\textsuperscript{41} came up with his Rough set theory. But there is one theory which is relatively new and having a lot of potential for being a major tool for modeling uncertainty, is the theory of Soft sets. In the year 1999, Russian mathematician Molodtsov [Molodtsov, 1999] initiated the theory of soft sets. His theory was primarily based on “parameterization of tools”. In dealing with uncertain situations, fuzzy set theory was perhaps the most appropriate theory till then. But the main difficulty with fuzzy sets is to frame a suitable membership function for a specific problem. The reason behind this is possibly the inadequacy of the parameterization tool of the theory. In his remarkable paper, Molodtsov proposed soft sets as a new mathematical tool for dealing with uncertainties and which is free from difficulties faced by previous fuzzy set theory.

A soft set is a classification of elements of the universe with respect to some given set of parameters. It has been shown that soft set is more general in nature and has more capabilities in handling uncertain information. Also a fuzzy set or a rough set can be considered as a special case of soft sets. Research involving soft sets and its application in various fields of science and technology are currently going on in a rapid pace.

This chapter deals with the theory of soft sets and is divided into several sections. In the first section some preliminary notions about soft sets and some operations on them are discussed. After the invention of soft sets in 1999, several generalizations of it came up in recent times. Here we have discussed a particular type of generalization of soft sets, called generalized fuzzy soft set in the second section. Section three describes an idea of soft mapping. Several algebraic structures like group, ring etc. have also extended in soft set settings. Some of these have been discussed in section four. A notion of soft topology has been discussed in section five. The notion of softness of a soft set or soft entropy has been discussed in section six. In the last section, some applications of soft sets have been shown.

THEORY OF SOFT SETS

A. Definitions

\textbf{Definition 1:} [Molodtsov, 1999] Let $U$ be a classical set of elements, called the universe and $E$ be a set of parameters, called the parameter set. Together they are often regarded as a soft universe. Members of the universe